Solution to Assignment 4
ECE 6601

1. \[ P_e = P_0 \cdot P_{10} + P_1 \cdot P_{01} \quad \text{(eqn 4.35)} \]
\[ P_e = P_0 \cdot \int_{-\infty}^{\infty} f_y(y10) \, dy + P_1 \cdot \int_{-\infty}^{\infty} f_y(y11) \, dy \quad \text{(eqns 4.28, 4.33)} \]
\[ \frac{\partial P_e}{\partial z} = P_0 \left( -f_y(210) \right) + P_1 \cdot f_y(211) = 0 \]
\[ \Rightarrow \frac{f_y(211)}{f_y(210)} = \frac{P_0}{P_1} \]

2. A sinc or a raised cosine function has a limited bandwidth; the NRZ pulse has an infinite bandwidth. Although small, the NRZ pulse creates some interference to adjacent bands. The NRZ pulse satisfies \[ \frac{1}{2} P(f - \pi P_b) = T_b. \]

3. The pulse already has zero-phase, which could be considered a type of linear phase. In general, to have a linear phase for the pulse \( p(t) \), you delay it by some time \( t_d \)
\[ p(t-t_d) = 2\pi C (2W(t-t_d)) \left( \frac{\cos(2\pi W(t-t_d))}{1 + 16 W^2 (t-t_d)^2} \right) \]
which would make \( P(f) \) flat. Therefore, the phase of the delayed pulse would be simply \(-2\pi ft_d\), which is linear in \( f \). The raised cosine function is infinite in length (in the time domain); therefore, it is truncated in practice. Hopefully, the truncation is designed to have a minimum impact on the frequency properties (Magnitude and Phase) and time properties (such as zero ISI). Moreover, this truncated pulse is delayed to make the system causal (from DSP, \( p(t) = 0 \) for \( t < 0 \))
4.a) Binary sequence \{b_k\} = 0 0 1 1 0 1 0 0 1
Two-level seq \{a_k\} = +1 -1 +1 -1 +1 -1 -1 +1
Doubly-binary coder output \{c_k\} = 0 0 2 0 0 0 0 -2 0
Estimate of \{a_k\}: \hat{a}_k = +1 -1 -1 -1 +1 -1 -1 +1
Receiver output \{\hat{b}_k\} = 0 0 1 0 1 0 0 1
The \(1\) is arbitrary, but has to be consistent.

b) \{c_k\} = 0 0 0 2 0 0 0 -2 0
\{\hat{a}_k\} = +1 -1 +1 -1 -1 +1 -1 +1
\{\hat{b}_k\} = 0 1 0 1 0 1 0 1
These estimates can be either +1 or -1.

5.a) \{\hat{b}_k\} = 0 0 1 1 0 1 0 0 1
\{\hat{d}_k\} = 1 -1 1 0 1 1 0 0 1
\{\hat{a}_k\} = +1 +1 -1 +1 +1 -1 -1 -1 +1
\{\hat{c}_k\} = 2 2 0 0 2 0 -2 -2 0
\{\hat{b}_k\} = 0 0 1 1 0 1 0 0 1
(4.76)
If \(|c_k| < 1\) \(\Rightarrow \hat{b}_k = 1\)
If \(|c_k| > 1\) \(\Rightarrow \hat{b}_k = 0\)

6) \{c_k\} = 2 0 0 0 2 0 -2 -2 0
\{\hat{b}_k\} = 0 1 1 1 0 1 0 0