Example 2.10 Construct the SFG for the block diagram shown in Fig.2-19.

![Block diagram for Example 2-10](image)

Fig.2-19 Block diagram for Example 2-10

The signal flow graph corresponding to the block diagram of Fig.2-19 is shown in Fig.2-20.

![Signal flow graph](image)

Fig.2-20 SFG equivalent to block diagram of Fig.2-19.

** Mason's Formula **

Before presenting the Mason's formula to determine the transfer function between the output and input nodes of a SFG, we define some new concepts. Consider the SFG of Fig.(2-20).
A forward path is a trajectory directed from the input node to the output node without passing twice through a node. In the SFG of Fig. 2-20 there are two forward paths with gains: \( G_1 G_4 G_5 \) and \( G_1 G_2 G_3 G_4 \).

A loop is a closed path following the arrows. There are four loops in Fig. 2-20 with gains: \(-H_1\) (self-loop), \(-G_2 H_2\), \(-G_2 G_3 G_4 H_3\) and \(-G_4 G_5 H_3\).

A nontouching graph to a forward path is the portion of the SFG that is not touching the forward path. For instance, the nontouching graph for forward path \( G_1 G_4 G_5 \) is the self-loop with gain \(-H_1\). Does the forward path \( G_1 G_2 G_3 G_4 \) have a nontouching graph?

Using Mason's formula, the transfer function \( \frac{C(s)}{R(s)} \) is given by

\[
\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{N} P_i \Delta_i}{\Delta}
\]

where

\[ N \quad : \quad \text{number of forward paths} \]
\[ P_i \quad : \quad \text{gain of the } i^{th} \text{ forward path} \]
\[ \Delta = 1 - \sum_{j} L_j^{'} + \sum_{k} L_k^{'} L_j^{'} - \sum_{q} L_q^{'''} + \ldots \]
\[ L_j^{'} \quad : \quad \text{gain of the } j^{th} \text{ loop} \]
\[ L_k^{'} \quad : \quad \text{gain product of two nontouching loops (no nodes in common)} \]
\[ L_q^{'''} \quad : \quad \text{gain product of three nontouching loops} \]
\[ \Delta_i \quad : \quad \Delta \text{ corresponding to that portion of the graph nontouching to path of gain } P_i \]

Example 2.11 For the SFG shown in Fig. 2-20, find the transfer function \( \frac{C(s)}{R(s)} \) using Mason's formula.

Step 1 There are two forward paths, as shown in Fig. 2-21(a),
\[ N = 2: \quad P_1 = G_1 G_2 G_3 G_4, \quad P_2 = G_1 G_4 G_5 \]

Step 2 There are four single loops (Fig. 2-21(b)),
\[ L_1^{'} = -H_1, \quad L_2^{'} = -G_2 H_2, \quad L_3^{'} = -G_2 G_3 G_4 H_3, \quad L_4^{'} = -G_4 G_5 H_3 \]

Step 3 As shown in Fig. 2-21(c), we have only two nontouching loops with product gain
\[ L_1^{''} = (-H_1)(-G_4 G_5 H_3) = G_4 G_5 H_1 H_3 \]
step 4 There is no group of three or more nontouching loops.

step 5 \( \Delta \) is given by

\[
\Delta = 1 + H_1 + G_2 H_2 + G_2 G_3 G_4 H_3 + G_4 G_5 H_3 + G_4 G_5 H_1 H_3
\]

step 6 There is no portion of graph nontouching to forward path number 1. Therefore, \( \Delta_1 = 1 \). The nontouching graph for forward path number 2 is the self-loop of gain \(-H_1\). Thus, \( \Delta_2 = 1 + H_1 \).

step 7 The closed-loop function is

\[
\frac{C(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{G_1(G_2 G_3 + G_5 + H_1 G_5) G_4}{1 + H_1 + G_2(H_2 + G_3 G_4 H_3) + G_4 G_5 H_3(1 + H_1)}
\]  
(2-86)

Fig.2-21  Forward paths and loops of SGF of Fig.2-20.

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