Lecture 2

Path Loss, Co-channel Interference, Link Budget
PROPAGATION OVER A FLAT SPECULAR SURFACE
• The length of the direct path is

\[ d_1 = \sqrt{d^2 + (h_b - h_m)^2} \]

and the length of the reflected path is

\[ d_2 = \sqrt{d^2 + (h_b + h_m)^2} \]

\[ d = \text{distance between mobile and base stations} \]
\[ h_b = \text{base station antenna height} \]
\[ h_m = \text{mobile station antenna height} \]

• Given that \( d \gg h_b h_m \), we have \( d_1 \approx d \) and \( d_2 \approx d \).

• The carrier phase difference between the direct and reflected paths is

\[ \phi_1 - \phi_2 = \frac{2\pi}{\lambda_c} (d_1 - d_2) \]
• Taking into account the phase difference, the received signal power is

\[ \mu \Omega_p = \Omega_t \left( \frac{\lambda_c}{4\pi d} \right)^2 G_T G_R \left| 1 + ae^{-jb}e^{j(\phi_2 - \phi_1)} \right|^2, \]

where \( a \) and \( b \) are the amplitude attenuation and phase change introduced by the flat reflecting surface.

• If we assume a perfect specular reflection, then \( a = 1 \) and \( b = \pi \) for small \( \theta \). Then

\[
\mu \Omega_p = \Omega_t \left( \frac{\lambda_c}{4\pi d} \right)^2 G_T G_R \left| 1 - e^{j(\frac{2\pi}{\lambda_c} \Delta d)} \right|^2
\]

\[
= \Omega_t \left( \frac{\lambda_c}{4\pi d} \right)^2 G_T G_R \sin^2 \left( \frac{2\pi}{\lambda_c} \Delta d \right)
\]

where \( \Delta d = (d_1 - d_2) \).

• Given that \( d \gg h_b \) and \( d \gg h_m \), and applying the approximation \( \sqrt{1 + x} \approx 1 + x/2 \) for small \( x \), we have

\[
\Delta d \approx \frac{2h_b h_m}{d}.
\]
Finally, the received envelope power is

\[ \mu_{\Omega_p} \approx 4\Omega_t \left( \frac{\lambda_c}{4\pi d} \right)^2 G_T G_R \sin^2 \left( \frac{2\pi h_b h_m}{\lambda_c d} \right) \]

Under the condition that \( d \gg h_b h_m \), the above reduces to

\[ \mu_{\Omega_p} \approx \Omega_t G_T G_R \left( \frac{h_b h_m}{d^2} \right)^2 \]

where we have invoked the small angle approximation \( \sin x \approx x \) for small \( x \).

Propagation over a plane reflecting surface differs from free space propagation in two respects

- it is not frequency dependent
- signal strength decays with the with the fourth power of the distance, rather than the square of the distance.
Propagation path loss $L_p \,(\text{dB})$ with distance over a flat reflecting surface; $h_b = 7.5 \, \text{m}, \, h_m = 1.5 \, \text{m}, \, f_c = 1800 \, \text{MHz}$.

$$L_p = \left[ \left( \frac{\lambda_c}{4\pi d} \right)^2 4 \sin^2 \left( \frac{2\pi h_b h_m}{\lambda_c d} \right) \right]^{-1}$$
In reality, the earth’s surface is curved and rough, and the signal strength typically decays with the inverse $\beta$ power of the distance, and the received power is

$$\Omega_p = k \frac{\Omega_t}{d^{\beta}}$$

where $k$ is a constant of proportionality. Expressed in units of dBm, the received power is

$$\Omega_p \text{ (dBm)} = 10\log_{10}(k) + \Omega_t \text{ (dBm)} - 10\beta\log_{10}(d)$$

$\beta$ is called the path loss exponent. Typical values of $\beta$ are have been determined by empirical measurements for a variety of areas

<table>
<thead>
<tr>
<th>Terrain</th>
<th>$\beta$</th>
</tr>
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<tbody>
<tr>
<td>Free Space</td>
<td>2</td>
</tr>
<tr>
<td>Open Area</td>
<td>4.35</td>
</tr>
<tr>
<td>North American Suburban</td>
<td>3.84</td>
</tr>
<tr>
<td>North American Urban (Philadelphia)</td>
<td>3.68</td>
</tr>
<tr>
<td>North American Urban (Newark)</td>
<td>4.31</td>
</tr>
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<td>Japanese Urban (Tokyo)</td>
<td>3.05</td>
</tr>
</tbody>
</table>
Co-channel Interference

Worst case co-channel interference on the forward channel.
Worst Case Co-Channel Interference

- There are six co-channel base-stations, two at distance $D - R$, two at distance $D$, and two at distance of $D + R$.

- The worst case carrier-to-interference ratio is

$$\Lambda = \frac{1}{2} \frac{R^{-\beta}}{(D - R)^{-\beta} + D^{-\beta} + (D + R)^{-\beta}}$$

$$= \frac{1}{2} \frac{1}{(\frac{D}{R} - 1)^{-\beta} + (\frac{D}{R})^{-\beta} + (\frac{D}{R} + 1)^{-\beta}}$$

$$= \frac{1}{2} \frac{1}{(\sqrt{3N} - 1)^{-\beta} + (\sqrt{3N})^{-\beta} + (\sqrt{3N} + 1)^{-\beta}}$$

- Hence, for $\beta = 3.5$

$$\Lambda_{(dB)} = \begin{cases} 14.3 \text{ dB} & \text{for } N = 7 \\ 9.2 \text{ dB} & \text{for } N = 4 \\ 6.3 \text{ dB} & \text{for } N = 3 \end{cases}$$

- Shadows will introduce variations in the worst case $C/I$. 
Worst case co-channel interference on the forward channel with 120° cell sectoring.
• 120° cell sectoring reduces the number of co-channel base stations from six to two. The co-channel base stations are at distances $D$ and $D + 0.7R$.

• The carrier-to-interference ratio becomes

$$\Lambda = \frac{R^{-\beta}}{D^{-\beta} + (D + 0.7R)^{-\beta}}$$

$$= \frac{1}{(\frac{D}{R})^{-\beta} + (\frac{D}{R} + 0.7)^{-\beta}}$$

$$= \frac{1}{(\sqrt{3N})^{-\beta} + (\sqrt{3N} + 0.7)^{-\beta}}$$

• Hence

$$\Lambda_{(dB)} = \begin{cases} 
21.1 \text{ dB} & \text{for } N = 7 \\
17.1 \text{ dB} & \text{for } N = 4 \\
15.0 \text{ dB} & \text{for } N = 3 
\end{cases}$$

• For $N = 7$, 120° cell sectoring yields a 6.8 dB C/I improvement over omni-cells.

• The minimum allowable cluster size is determined by the minimum C/I requirement of the radio receiver. For example, if the radio receiver can operate at $\Lambda = 15.0$ dB, then a 3/9 reuse cluster can be used (3/9 means 3 cells or 9 sectors per cluster).
Receiver Sensitivity

• Receiver sensitivity refers to the ability of the receiver to detect radio signals. We would like our radio receivers to be as sensitive as possible.

• Radio receivers must detect radio waves in the presence of noise.
  – External noise sources include atmospheric noise (e.g., lightning strikes), galactic noise, man made noise (e.g., automobile ignition noise).
  – Internal noise sources include thermal noise.

• The ratio of the desired signal power to thermal noise power before detection is commonly called the carrier-to-noise ratio, $\Gamma$.

• The parameter $\Gamma$ is a function of the communication link parameters including transmitted power (or effective isotropic radiated power (EIRP)), path loss, receiver antenna gain, and the effective input-noise temperature of the receiving system.

• The formula that relates the link parameters to $\Gamma$ is called the link budget.
Link Budget

- The link budget can be expressed in terms of the following parameters:

  \[ \Omega_t = \text{transmitted carrier power} \]
  \[ G_T = \text{transmitter antenna gain} \]
  \[ L_p = \text{path loss} \]
  \[ G_R = \text{receiver antenna gain} \]
  \[ \Omega_p = \text{received signal power} \]
  \[ E_c = \text{received energy per modulated symbol} \]
  \[ T_o = \text{receiving system noise temperature in degrees Kelvin} \]
  \[ B_w = \text{receiver noise equivalent bandwidth} \]
  \[ N_o = \text{white noise power spectral density} \]
  \[ R_c = \text{modulated symbol rate} \]
  \[ k = 1.38 \times 10^{-23} = \text{Boltzmann’s constant} \]
  \[ F = \text{noise figure, typically about 3 dB} \]
  \[ L_{Rx} = \text{receiver implementation losses} \]
  \[ L_I = \text{losses due to system load (interference)} \]
  \[ M_{shad} = \text{shadow margin} \]
  \[ G_{HO} = \text{handoff gain} \]
  \[ S_{RX} = \text{receiver sensitivity} \]
• The effective received carrier power is
\[ \Omega_p = \frac{\Omega_t G_T G_R}{L_{Rx}L_p} . \]

• The total input noise power to the receiver is
\[ N = kT_o B_w F \]

• Very often the following \( kT_o \) value at room temperature of 17 °C (290 °K) is used \( kT_o = -174 \) dBm/Hz,

• The received carrier-to-noise ratio defines the link budget
\[ \Gamma = \frac{\Omega_p}{N} = \frac{\Omega_t G_T G_R}{kT_o B_w F L_{Rx}L_p} . \]

• The carrier-to-noise ratio, \( \Gamma \), and modulated symbol energy-to-noise ratio, \( E_c/N_o \), are related as follows
\[ \frac{E_c}{N_o} = \Gamma \times \frac{B_w}{R_c} . \]

• Hence, we can rewrite the link budget as
\[ \frac{E_c}{N_o} = \frac{\Omega_t G_T G_R}{kT_o R_c F L_{Rx}L_p} . \]
• Converting into decibel units gives

\[
\frac{E_c}{N_0} \text{(dB)} = \Omega_t \text{ (dBm)} + G_T \text{ (dB)} + G_R \text{ (dB)} - kT_o \text{ (dBm/Hz)} - R_c \text{ (dB Hz)} - F \text{ (dB)} - L_{R_x} \text{ (dB)} - L_p \text{ (dB)}. \quad (1)
\]

• The receiver sensitivity is defined as

\[
S_{R_x} = L_{R_x}kT_oF\left(\frac{E_c}{N_0}\right)R_c
\]

or converting to decibel units

\[
S_{R_x} \text{ (dBm)} = L_{R_x} \text{ (dB)} + kT_o \text{ (dBm/Hz)} + F \text{ (dB)} + \frac{E_c}{N_0} \text{ (dB)} + R_c \text{ (dB Hz)}. \]

• All parameters are usually fixed except for \(\frac{E_c}{N_0}\). The receiver sensitivity (in dBm) is determined by the minimum acceptable \(\frac{E_c}{N_0}\).

• Substituting the determined receiver sensitivity \(S_{R_x} \text{ (dBm)}\) into (1) and solving for \(L_p \text{ (dB)}\) gives the maximum allowable path loss

\[
L_{\text{max}} \text{ (dB)} = \Omega_t \text{ (dBm)} + G_T \text{ (dB)} + G_R \text{ (dB)} - S_{R_x} \text{ (dBm)}. \]