ECE6604
PERSONAL & MOBILE COMMUNICATIONS

Week 4

Envelope Correlation
Space-time Correlation
Autocorrelation of a Bandpass Random Process

• Consider again the received band-pass random process

\[ r(t) = g_I(t) \cos 2\pi f_c t - g_Q(t) \sin 2\pi f_c t \]

where

\[ g_I(t) = \sum_{n=1}^{N} C_n \cos \phi_n(t) \]
\[ g_Q(t) = \sum_{n=1}^{N} C_n \sin \phi_n(t) \]

• Assuming that \( r(t) \) is wide-sense stationary, the autocorrelation of \( r(t) \) is

\[ \phi_{rr}(\tau) = \mathbb{E}[r(t)r(t+\tau)] \]
\[ = \mathbb{E}[g_I(t)g_I(t+\tau)] \cos 2\pi f_c \tau + \mathbb{E}[g_Q(t)g_I(t+\tau)] \sin 2\pi f_c \tau \]
\[ = \phi_{g_ig_I}(\tau) \cos 2\pi f_c \tau - \phi_{g_Ig_Q}(\tau) \sin 2\pi f_c \tau \]

where \( \mathbb{E}[\cdot] \) is the ensemble average operator, and

\[ \phi_{g_ig_I}(\tau) \triangleq \mathbb{E}[g_I(t)g_I(t+\tau)] \]
\[ \phi_{g_Ig_Q}(\tau) \triangleq \mathbb{E}[g_I(t)g_Q(t+\tau)] \].

Note that the wide-sense stationarity of \( r(t) \) imposes the condition

\[ \phi_{g_ig_I}(\tau) = \phi_{g_Qg_Q}(\tau) \]
\[ \phi_{g_Ig_Q}(\tau) = -\phi_{g_Qg_I}(\tau) \].
Auto- and Cross-correlation of Quadrature Components

- The phases $\phi_n(t)$ are statistically independent random variables at any time $t$, uniformly distributed over the interval $[-\pi, \pi]$.

- The azimuth angles of arrival, $\theta_n$ are all independent due to the random placement of scatterers. Also, in the limit $N \to \infty$, the discrete azimuth angles of arrival $\theta_n$ can be replaced by a continuous random variable $\theta$ having the probability density function $p(\theta)$.

- By using the above properties, the auto- and cross-correlation functions can be obtained as follows:

$$
\phi_{g_1g_1}(\tau) = \phi_{g_Qg_Q}(\tau) = \lim_{N \to \infty} E_{\tau, \theta, \phi}[g_I(t)g_I(t+\tau)] = \frac{\Omega_p}{2} E_\theta[\cos(2\pi f_m \tau \cos \theta)]
$$

$$
\phi_{g_1g_Q}(\tau) = -\phi_{g_Qg_I}(\tau) = \lim_{N \to \infty} E_{\tau, \theta, \phi}[g_I(t)g_Q(t+\tau)] = \frac{\Omega_p}{2} E_\theta[\sin(2\pi f_m \tau \cos \theta)]
$$

$$
\tau = (\tau_1, \tau_2, \ldots, \tau_N)
$$

$$
\theta = (\theta_1, \theta_2, \ldots, \theta_N)
$$

$$
\phi = (\phi_1, \phi_2, \ldots, \phi_N)
$$

$$
\Omega_p = E[g_I^2(t)] + E[g_Q^2(t)] = \sum_{n=1}^{N} C_n^2
$$

and $\Omega_p$ is the total received envelope power.
2-D Isotropic Scattering

- Evaluation of the expectations for the auto- and cross-correlation functions requires the azimuth distribution of arriving plane waves $p(\theta)$, and the receiver antenna gain pattern $G(\theta)$, as a function of the azimuth angle $\theta$.

- With 2-D isotropic scattering, the plane waves are confined to the $x-y$ plane and arrive uniformly distributed angle of incidence, i.e.,

$$p(\theta) = \frac{1}{2\pi}, \quad -\pi \leq \theta \leq \pi$$

- With 2-D isotropic scattering and an isotropic receiver antenna with gain $G(\theta) = 1, \theta \in [-\pi, \pi)$, the auto- and cross-correlation functions become

$$\phi_{g_Ig_I}(\tau) = \frac{\Omega_p}{2} J_0(2\pi f_m \tau)$$

$$\phi_{g_Ig_Q}(\tau) = 0$$

where

$$J_0(x) = \frac{1}{\pi} \int_{0}^{\pi} \cos(x \cos \theta) d\theta$$

is the zero-order Bessel function of the first kind.
Normalized autocorrelation function of the quadrature components of the received complex envelope with 2-D isotropic scattering and an isotropic receiver antenna.
- The autocorrelation function and power spectral density (psd) are Fourier transform pairs.

\[
S_{gg}(f) = \int_{-\infty}^{\infty} \phi_{gg}(\tau)e^{-j2\pi f \tau} d\tau
\]

\[
\phi_{gg}(\tau) = \int_{-\infty}^{\infty} S_{gg}(f)e^{j2\pi f \tau} df
\]

- The autocorrelation of the received complex envelope \(g(t) = g_I(t) + jg_Q(t)\) is

\[
\phi_{gg}(\tau) = \frac{1}{2}E[g^*(t)g(t+\tau)]
\]

\[
= \phi_{g_Ig_I}(\tau) + j\phi_{g_Ig_Q}(\tau)
\]

- The Fourier transform of \(\phi_{gg}(\tau)\) gives the Doppler psd

\[
S_{gg}(f) = S_{g_Ig_I}(f) + jS_{g_Ig_Q}(f)
\]

Sometimes \(S_{gg}(f)\) is just called the “\textbf{Doppler spectrum}.”
We can also relate the power spectrum of the complex envelope $g(t)$ to that of the band-pass process $r(t)$. We have

$$\phi_{rr}(\tau) = \text{Re} \left[ \phi_{gg}(\tau)e^{j2\pi f_c \tau} \right].$$

By using the identity

$$\text{Re} [z] = \frac{z + z^*}{2}$$

and the property $\phi_{gg}(\tau) = \phi_{gg}^*(-\tau)$, it follows that the band-pass Doppler psd is

$$S_{rr}(f) = \frac{1}{2} \left[ S_{gg}(f - f_c) + S_{gg}(-f - f_c) \right].$$

Since $\phi_{gg}(\tau) = \phi_{gg}^*(-\tau)$, the Doppler spectrum $S_{gg}(f)$ is always a real-valued function of frequency, but not necessarily even. However, the band-pass Doppler spectrum $S_{rr}(f)$ is always real-valued and even.
Isotropic Scattering

- For 2-D isotropic scattering, the psd and cross psd of $g_I(t)$ and $g_Q(t)$ are

$$S_{g_Ig_I}(f) = \mathcal{F}[\phi_{g_Ig_I}(\tau)] = \begin{cases} \frac{\Omega_p}{2\pi f_m} \frac{1}{\sqrt{1-(f/f_m)^2}}, & |f| \leq f_m \\ 0, & \text{otherwise} \end{cases}$$

$$S_{g_Ig_Q}(f) = 0$$

Normalized psd of the quadrature components of the received complex envelope with 2-D isotropic scattering and an isotropic receiver antenna. Sometimes this is called the CLASSICAL Doppler power spectrum.
Non-isotropic Scattering – Rician Fading

• Suppose that the propagation environment consisting of a strong specular component plus a scatter component. The azimuth distribution $p(\theta)$ might have the form

$$p(\theta) = \frac{1}{K+1} \hat{p}(\theta) + \frac{K}{K+1} \delta(\theta - \theta_0)$$

where $\hat{p}(\theta)$ is the continuous AoA distribution of the scatter component, $\theta_0$ is the AoA of the specular component, and $K$ is the ratio of the received specular to scattered power.

• One such scattering environment, assumes that the scatter component exhibits 2-D isotropic scattering, i.e., $\hat{p}(\theta) = 1/(2\pi), \theta \in [-\pi, \pi]$.

• The correlation functions $\phi_{g_1g_1}(\tau)$ and $\phi_{g_1g_Q}(\tau)$ are

$$\phi_{g_1g_1}(\tau) = \frac{1}{K+1} \frac{\Omega_p}{2} J_0(2\pi f_m \tau) + \frac{K}{K+1} \frac{\Omega_p}{2} \cos(2\pi f_m \tau \cos \theta_0)$$

$$\phi_{g_1g_Q}(\tau) = \frac{K}{K+1} \frac{\Omega_p}{2} \sin(2\pi f_m \tau \cos \theta_0) .$$
Plot of $p(\theta)$ vs. $\theta$ with 2-D isotropic scattering plus a LoS or specular component arriving at angle $\theta_0 = \pi/2$. 

![Graph showing $K=1$ and $\theta_0 = \pi/2$](image-url)
- The azimuth distribution

\[ p(\theta) = \frac{1}{K + 1} \hat{p}(\theta) + \frac{K}{K + 1} \delta(\theta - \theta_0) \]

yields a complex envelope having a Doppler spectrum of the form

\[ S_{gg}(f) = \frac{1}{K + 1} S_{gg}^c(f) + \frac{K}{K + 1} S_{gg}^d(f) \]  \hspace{1cm} (1)

where \( S_{gg}^d(f) \) is the discrete portion of the Doppler spectrum due to the specular component and \( S_{gg}^c(f) \) is the continuous portion of the Doppler spectrum due to the scatter component.

- For the case when \( \hat{p}(\theta) = 1/(2\pi), \theta \in [-\pi, \pi] \), the power spectrum of \( g(t) = g_I(t) + jg_Q(t) \) is

\[
S_{gg}(f) = \begin{cases} 
\frac{1}{K+1} \cdot \frac{\Omega_p}{2\pi f_m} \sqrt{\frac{1}{1-(f/f_m)^2}} 
+ \frac{K}{K+1} \cdot \Omega_p \delta(f - f_m \cos \theta_0) & 0 \leq |f| \leq f_m \\
0 & \text{otherwise}
\end{cases}
\]

- Note the discrete tone at frequency \( f_c + f_m \cos \theta_0 \) due to the line-of-sight or specular component arriving from angle \( \theta_0 \).
Non-isotropic scattering – Other Cases

- Sometimes the azimuth distribution $p(\theta)$ may not be uniform, a condition commonly called non-isotropic scattering. Several distributions have been suggested to model non-isotropic scattering.

- One possibility is the Gaussian distribution

$$p(\theta) = \frac{1}{\sqrt{2\pi}\sigma_S} \exp \left\{ -\frac{(\theta - \mu)^2}{2\sigma_S^2} \right\}$$

where $\mu$ is the mean AoA, and $\sigma_S$ is the rms AoA spread.

- Another possibility is the von Mises distribution

$$p(\theta) = \frac{1}{2\pi I_0(k)} \exp \left[ k \cos(\theta - \mu) \right] ,$$

where $\theta \in [-\pi, \pi)$, $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind, $\mu \in [-\pi, \pi)$ is the mean AoA, and $k$ controls the spread of scatterers around the mean.
Plot of $p(\theta)$ vs. $\theta$ for the von Mises distribution with a mean angle-to-arrival $\mu = \pi/2$. 
The Doppler spectrum can be derived by using a different approach that is sometimes very useful because it can avoid the need to evaluate integrals. The Doppler spectrum can be expressed as

\[ S_{gg}(f)|df| = \frac{\Omega_p}{2} (G(\theta)p(\theta) + G(-\theta)p(-\theta))|d\theta| \ . \]

The Doppler frequency associated with the incident plane wave arriving at angle \( \theta \) is

\[ f = f_m \cos(\theta) \ , \]

and, hence,

\[ |df| = f_m| - \sin(\theta)d\theta| = \sqrt{f_m^2 - f^2} |d\theta| \ . \]

Therefore,

\[ S_{gg}(f) = \frac{\Omega_p/2}{\sqrt{f_m^2 - f^2}} (G(\theta)p(\theta) + G(-\theta)p(-\theta)) \ , \]

where

\[ \theta = \cos^{-1}(f / f_m) \ . \]

Hence, if \( p(\theta) \) and \( G(\theta) \) are known, the Doppler spectrum can be easily calculated. For example, with 2-D isotropic scattering and an isotropic antenna \( G(\theta)p(\theta) = 1/(2\pi) \).
Many mobile radio systems use antenna diversity, where spatially separated receiver antennas provide multiple faded replicas of the same information bearing signal.

The spatial decorrelation of the channel tell us the required spatial separation between antenna elements so that they will be “sufficiently” decorrelated.

Sometimes it is desirable to simultaneously characterize both the spatial and temporal correlation characteristics of the channel, e.g., when using space-time coding, fading process. This can be described by the space-time correlation function.

To obtain the spatial or space-time correlation functions, we must specify some kind of radio scattering geometry.
Single-ring scattering model for NLoS propagation on the forward link of a cellular system. The MS is surrounded by a scattering ring of radius $R$ and is at distance $D$ from the BS, where $R \ll D$. 
Model Parameters

- $O_B$: base station location
- $O_M$: mobile station location
- $D$: LoS distance from base station to mobile station
- $R$: scattering radius
- $\gamma_M$: mobile station moving direction w.r.t $x$-axis
- $v$: mobile station speed
- $\theta_M$: mobile station array orientation w.r.t. $x$-axis
- $A_M^{(i)}$: location of $i$th mobile station antenna element
- $\delta_M$: distance between mobile station antenna elements
- $S_M^{(n)}$: location of $n$th scatterer.
- $\alpha_M^{(n)}$: angle of arrival from the $n$th scatterer.
- $\epsilon_n$: distance $O_B - S_M^{(n)}$
- $\epsilon_{ni}$: distance $S_M^{(n)} - A_M^{(i)}$. 
Received Complex Envelope

• The channel from $O_B$ to $A_M^{(q)}$ has the complex envelope

$$g_q(t) = \sum_{n=1}^{N} C_n e^{j\phi_n - j2\pi(\epsilon_n + \epsilon_{nq})/\lambda_c e^{j2\pi f_m t \cos(\alpha_{M}^{(n)} - \gamma_M)}}, \ q = 1, 2$$

where $\epsilon_n$ and $\epsilon_{nq}$ denote the distances $O_B - S_M^{(n)}$ and $S_M^{(n)} - A_M^{(q)}$, $q = 1, 2$, respectively, and $\phi_n$ is a uniform random phase on the interval $[-\pi, \pi)$.

• From the Law of Cosines, the distances $\epsilon_n$ and $\epsilon_{nq}$ can be expressed as a function of the angle-of-arrival $\alpha_{M}^{(n)}$ as follows:

$$\epsilon_n^2 = D^2 + R^2 + 2DR \cos \alpha_M^{(n)} \quad \text{Note sign change since the angle is } \pi - \alpha_M^{(n)}$$

$$\epsilon_{nq}^2 = [(1.5 - q)\delta_M]^2 + R^2 - 2(1.5 - q)\delta_M R \cos(\alpha_M^{(n)} - \theta_M), \ q = 1, 2.$$  

• Assuming that $R/D \ll 1$ (local scattering), $\delta_M \ll R$ and $\sqrt{1 \pm x} \approx 1 \pm x/2$ for small $x$, we have

$$\epsilon_n \approx D + R \cos \alpha_M^{(n)}$$

$$\epsilon_{nq} \approx R - (1.5 - q)\delta_M \cos(\alpha_M^{(n)} - \theta_M), \ q = 1, 2.$$  

• Hence,

$$g_q(t) = \sum_{n=1}^{N} C_n e^{j\phi_n - j2\pi\left(D+R \cos \alpha_M^{(n)}+R-(1.5-q)\delta_M \cos(\alpha_M^{(n)}-\theta_M)\right)/\lambda_c}$$

$$\times e^{j2\pi f_m t \cos(\alpha_M^{(n)} - \gamma_M)}}, \ q = 1, 2.$$
• The space-time correlation function between the two complex faded envelopes $g_1(t)$ and $g_2(t)$ is

$$
\phi_{g_1,g_2}(\delta_M, \tau) = \frac{1}{2} \mathbb{E} [g_1(t)g_2(t + \tau)^*]
$$

• The space-time correlation function between $g_1(t)$ and $g_2(t)$ can be written as

$$
\phi_{g_1,g_2}(\delta_M, \tau) = \Omega p^2 \sum_{n=1}^{N} \mathbb{E} \left[ e^{j2\pi(\delta_M/\lambda_c)\cos(\alpha_M^{(n)} - \theta_M)} e^{-j2\pi f_m \tau \cos(\alpha_M^{(n)} - \gamma_M)} \right].
$$

• Since the number of scatters is infinite, the discrete angles-of-arrival $\alpha_M^{(n)}$ can be replaced with a continuous random variable $\alpha_M$ with probability density function $p(\alpha_M)$.

• Hence, the space-time correlation function becomes

$$
\phi_{g_1,g_2}(\delta_M, \tau) = \frac{\Omega p}{2N} \int_{0}^{2\pi} e^{j b \cos(\alpha_M - \theta_M)} e^{-ja \cos(\alpha_M - \gamma_M)} p(\alpha_M) d\alpha_M
$$

where $a = 2\pi f_m \tau$ and $b = 2\pi \delta_M / \lambda_c$. 
2-D Isotropic Scattering

- For the case of 2-D isotropic scattering with an isotropic receive antennas, \( p(\alpha_M) = 1/(2\pi), -\pi \leq \alpha_M \leq \pi \), and the space-time correlation function becomes
  \[
  \phi_{g_1,g_2}(\delta_M, \tau) = \frac{\Omega p}{2} J_0 \left( \sqrt{a^2 + b^2 - 2ab \cos(\theta_M - \gamma_M)} \right).
  \]

- The spatial and temporal correlation functions can be obtained by setting \( \tau = 0 \) and \( \delta_M = 0 \), respectively. This gives
  \[
  \phi_{g_1,g_2}(\delta_M) = \phi_{g_1,g_2}(\delta_M, 0) = \frac{\Omega p}{2} J_0(2\pi \delta_M/\lambda_c)
  \]
  \[
  \phi_{gg}(\tau) = \phi_{g_1,g_2}(0, \tau) = \frac{\Omega p}{2} J_0(2\pi f_m \tau)
  \]

- Finally, we note that
  \[
  f_m \tau = \frac{v \cdot \tau}{\lambda_c} = \frac{\delta_M}{\lambda_c}
  \]

For this scattering environment, the normalized time \( f_m \tau \) is equivalent to the normalized distance \( \delta_M/\lambda_c \).

- The antenna branches are uncorrelated if they are separated by \( \delta_M \approx 0.5\lambda_c \).
Temporal and spatial correlation functions at the MS with 2-D isotropic scattering and an isotropic receiver antenna. Note that $f_m \tau = \delta_M / \lambda_c$. 
Single-ring scattering model for NLoS propagation on the reverse link of a cellular system. The MS is surrounded by a scattering ring of radius $R$ and is at distance $D$ from the BS, where $R \ll D$. 

**Spatial Correlation at the Base Station**
Model Parameters

- $O_B$: base station location
- $O_M$: mobile station location
- $D$: LoS distance from base station to mobile station
- $R$: scattering radius
- $\gamma_M$: mobile station moving direction w.r.t $x$-axis
- $v$: mobile station speed
- $\theta_B$: base station array orientation w.r.t. $x$-axis
- $A_B^{(i)}$: location of $i$th base station antenna element
- $\delta_B$: distance between mobile station antenna elements
- $S_M^{(m)}$: location of $m$th scatterer.
- $\alpha_M^{(m)}$: angle of departure to the $n$th scatterer.
- $\epsilon_m$: distance $S_M^{(m)} - O_B$.
- $\epsilon_{mi}$: distance $S_M^{(m)} - A_B^{(i)}$. 
**Received Complex Envelope**

- The channel from $O_M$ to $A_B^{(q)}$ has the complex envelope

$$g_q(t) = \sum_{m=1}^{N} C_m e^{j\phi_m} - j2\pi(R+\epsilon_{mq})/\lambda c e^{j2\pi f_m t \cos(\alpha_M^{(m)} - \gamma_M)}, \quad q = 1, 2$$

where $\epsilon_{mq}$ denote the distance $S_M^{(m)} - A_B^{(q)}$, $q = 1, 2$, and $\phi_m$ is a uniform random phase on $(-\pi, \pi]$. To proceed further, we need to express $\epsilon_{mq}$ as a function of $\alpha_M^{(m)}$.

- Applying the Law of Cosines to the triangle $\Delta S_M^{(m)} O_B A_B^{(q)}$, the distance $\epsilon_{mq}$ can be expressed as a function of the angle $\theta_B^{(m)} - \theta_B$ as follows:

$$\epsilon_{mq}^2 = [(1.5 - q)\delta_B]^2 + \epsilon_m^2 - 2(1.5 - q)\delta_B \epsilon_m \cos(\theta_B^{(m)} - \theta_B), \quad q = 1, 2.$$  \hspace{1cm} (3)

where $\epsilon_m$ is the distance $S_M^{(m)} - O_B$.

- By applying the Law of Sines to the triangle $\Delta O_M S_M^{(m)} O_B$ we obtain following identity

$$\frac{\epsilon_m}{\sin \alpha_M^{(m)}} = \frac{R}{\sin(\pi - \theta_B^{(m)})} = \frac{D}{\sin \left(\pi - \alpha_M^{(m)} - (\pi - \theta_B^{(m)})\right)}.$$
Received Complex Envelope

• Since the angle $\pi - \theta_B^{(m)}$ is small, we can apply the small angle approximations $\sin x \approx x$ and $\cos x \approx 1$ for small $x$, to the second equality in the above identity. This gives

$$\frac{R}{(\pi - \theta_B^{(m)})} \approx \frac{D}{\sin(\pi - \alpha_{M}^{(m)})}$$

or

$$(\pi - \theta_B^{(m)}) \approx (R/D) \sin(\pi - \alpha_{M}^{(m)}) \, .$$

• It follows that the cosine term in (2) becomes

$$\cos(\theta_B^{(m)} - \theta_B) = \cos(\pi - \theta_B - (\pi - \theta_B^{(m)}))$$

$$= \cos(\pi - \theta_B) \cos(\pi - \theta_B^{(m)}) + \sin(\pi - \theta_B) \sin(\pi - \theta_B^{(m)})$$

$$\approx \cos(\pi - \theta_B) + \sin(\pi - \theta_B)(R/D) \sin(\pi - \alpha_{M}^{(m)})$$

$$= - \cos(\theta_B) + (R/D) \sin(\theta_B) \sin(\alpha_{M}^{(m)})$$

(4)

• Using the approximation in (4) in (2), along with $\delta_B/\epsilon_m \ll 1$, gives

$$\epsilon_{mq}^2 \approx \epsilon_m^2 \left[ 1 - 2(1.5 - q) \frac{\delta_B}{\epsilon_m} \left[ (R/D) \sin(\theta_B) \sin(\alpha_{M}^{(m)}) - \cos(\theta_B) \right] \right] \, .$$
Received Complex Envelope

- Applying the approximation $\sqrt{1 \pm x} \approx 1 \pm x/2$ for small $x$, we have
  \[ \epsilon_{mq} \approx \epsilon_m - (1.5 - q)\delta_B \left[ (R/D) \sin(\theta_B) \sin(\alpha_M^{(m)}) - \cos(\theta_B) \right] . \] (5)

- Applying the Law of Cosines to the triangle $\triangle O_M S_M^{(m)} O_B$ we have
  \[ \epsilon_m^2 = D^2 + R^2 - 2DR \cos(\alpha_M^{(m)}) \]
  \[ \approx D^2 \left[ 1 - 2(R/D) \cos(\alpha_M^{(m)}) \right] , \]
  and again using the approximation $\sqrt{1 \pm x} \approx 1 \pm x/2$ for small $x$, we have
  \[ \epsilon_m \approx D - R \cos(\alpha_M^{(m)}) \] (6)

- Finally, using (5) in (4) gives
  \[ \epsilon_{mq} \approx D - R \cos(\alpha_M^{(m)}) - (1.5 - q)\delta_B \left[ (R/D) \sin(\theta_B) \sin(\alpha_M^{(m)}) - \cos(\theta_B) \right] . \] (7)

- Substituting (7) into (2) gives the result
  \[ g_q(t) = \sum_{m=1}^{N} C_m e^{j\phi_m + j2\pi f_m t \cos(\alpha_M^{(m)} - \gamma_M)} \]
  \[ \times e^{-j2\pi \left[ R+D-R \cos(\alpha_M^{(m)}) - (1.5-q)\delta_B \left[ (R/D) \sin(\theta_B) \sin(\alpha_M^{(m)}) - \cos(\theta_B) \right] \right]} / \lambda_c , \]
  which no longer depends on the $\epsilon_{mq}$ and is a function of the angle of departure $\alpha_M^{(m)}$.  

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Space-time Correlation Function

- The space-time correlation function between the two complex faded envelopes \( g_1(t) \) and \( g_2(t) \) at the BS is once again given by

\[
\phi_{g_1,g_2}(\delta_B, \tau) = \frac{1}{2} \mathbb{E}[g_1(t)g_2(t + \tau)^*]
\]

Using (8), the space-time correlation function between \( g_1(t) \) and \( g_2(t) \) can be written as

\[
\phi_{g_1,g_2}(\delta_B, \tau) = \Omega \frac{\Omega_p}{2N} \sum_{m=1}^{\infty} \mathbb{E}\left[ e^{j2\pi(\delta_B/\lambda_c)}\left( (R/D) \sin(\theta_B) \sin(\alpha_M^{(m)}) - \cos(\theta_B) \right) \right] \\
\times e^{-j2\pi f_m \tau \cos(\alpha_M^{(m)} - \gamma_M)}.
\]

- Since the number of scatters around the MS is infinite, the discrete angles-of-departure \( \alpha_M^{(m)} \) can be replaced with a continuous random variable \( \alpha_M \) with probability density function \( p(\alpha_M) \).

- Hence, the space-time correlation function becomes.

\[
\phi_{g_1,g_2}(\delta_B, \tau) = \frac{\Omega_p}{\lambda_c} \int_{-\pi}^{\pi} e^{-ja \cos(\alpha_M - \gamma_M)} e^{j[b((R/D) \sin(\theta_B) \sin(\alpha_M) - \cos(\theta_B))]d\alpha_M},
\]

where \( a = 2\pi f_m \tau \) and \( b = 2\pi \delta_B/\lambda_c \).
2-D Isotropic Scattering

- For the case of 2-D isotropic scattering with an isotropic MS transmit antenna, \( p(\alpha_M) = 1/(2\pi), -\pi \leq \alpha_M \leq \pi \), and the space-time correlation function becomes

\[
\phi_{g_1,g_2}(\delta_B, \tau) = \frac{\Omega_p}{2} e^{-jb\cos(\theta_B)} \times J_0 \left( \sqrt{a^2 + b^2(R/D)^2 \sin^2(\theta_B)} - 2ab(R/D) \sin(\theta_B) \sin(\gamma_M) \right).
\]

- The spatial and temporal correlation functions can be obtained by setting \( \tau = 0 \) and \( \delta_B = 0 \), respectively.

- The temporal correlation function \( \phi_{gg}(\tau) = \phi_{g_1,g_2}(0, \tau) = \frac{\Omega_p}{2} J_0(2\pi f_m \tau) \) which matches our result for the received signal at a mobile station.

- The spatial correlation function is

\[
\phi_{g_1,g_2}(\delta_B) = \phi_{g_1,g_2}(\delta_B, 0) = \frac{\Omega_p}{2} e^{-jb\cos(\theta_B)} J_0(b(R/D) \sin(\theta_B))
= \frac{\Omega_p}{2} e^{-jb\cos(\theta_B)} J_0 \left( \frac{2\pi \delta_B}{\lambda_c} (R/D) \sin(\theta_B) \right)
\]

- Observe that a much greater spatial separation is required to achieve a given degree of envelope decorrelation at the BS as compared to the MS. This can be readily seen by the term \( R/D \ll 1 \) in the argument of the Bessel function.
Envelope crosscorrelation magnitude at the base station for different base station antenna orientation angles, $\theta_B$; $D = 3000$ m, $R = 60$ m. Broadside base station antennas have the lowest crosscorrelation.
Envelope crosscorrelation magnitude at the base station for $\theta_B = \pi/3$ and various scattering radii, $R$; $D = 3000$ m. Smaller scattering radii will result in larger crosscorrelations.