EE6604
Personal & Mobile Communications

Week 8

Path Loss Models

Shadowing

Co-Channel Interference
Okumura-Hata Model

\[ L_p = \begin{cases} 
A + B \log_{10}(d) & \text{for urban area} \\
A + B \log_{10}(d) - C & \text{for suburban area} \\
A + B \log_{10}(d) - D & \text{for open area}
\end{cases} \]

where

\[ A = 69.55 + 26.16 \log_{10}(f_c) - 13.82 \log_{10}(h_b) - a(h_m) \]
\[ B = 44.9 - 6.55 \log_{10}(h_b) \]
\[ C = 5.4 + 2 \left[ \log_{10}(f_c/28) \right]^2 \]
\[ D = 40.94 + 4.78 \left[ \log_{10}(f_c) \right]^2 - 19.33 \log_{10}(f_c) \]

- Okumura and Hata’s model is in terms of
  - carrier frequency \( 150 \leq f_c \leq 1000 \) (MHz)
  - BS antenna height \( 30 \leq h_b \leq 200 \) (m)
  - MS antenna height \( 1 \leq h_m \leq 10 \) (m)
  - distance \( 1 \leq d \leq 20 \) (km) between the BS and MS.

- The model is known to be accurate to within 1 dB for distances ranging from 1 to 20 km.
● The parameter $a(h_m)$ is a “correction factor”

$$a(h_m) = \begin{cases} 
(1.1 \log_{10}(f_c) - 0.7) h_m - (1.56 \log_{10}(f_c) - 0.8) \\
8.28 (\log_{10}(1.54h_m))^2 - 1.1 & \text{for } f_c \leq 200 \text{ MHz} \\
3.2 (\log_{10}(11.75h_m))^2 - 4.97 & \text{for } f_c \geq 400 \text{ MHz} 
\end{cases}$$

for medium or small city

for large city
Path loss predicted by the Okumura-Hata model. Large city, $f_c = 900$ MHz, $h_b = 70$ m, $h_m = 1.5$ m.
CCIR Model

To account for varying degrees of urbanization, the CCIR (Comité International des Radio-Communication, now ITU-R) developed an empirical model for the path loss as:

\[ L_p \text{ (dB)} = A + B \log_{10}(d) - E \]

where \( A \) and \( B \) are defined in the Okumura-Hata model with \( a(h_m) \) being the medium or small city value.

The parameter \( E \) accounts for the degree of urbanization and is given by

\[ E = 30 - 25 \log_{10}(\% \text{ of area covered by buildings}) \]

where \( E = 0 \) when the area is covered by approximately 16% buildings.
Lee’s Area-to-area Model

• Lee’s area-to-area model is used to predict a path loss over flat terrain. If the actual terrain is not flat, e.g., hilly, there will be large prediction errors.

• Two parameters are required for Lee’s area-to-area model; the power at a 1 mile (1.6 km) point of interception, $\mu_{\Omega_p}(d_o)$, and the path-loss exponent, $\beta$.

• The received signal power at distance $d$ can be expressed as

$$\mu_{\Omega_p}(d) = \mu_{\Omega_p}(d_o) \left( \frac{d}{d_o} \right)^{-\beta} \left( \frac{f}{f_o} \right)^{-n} \alpha_0$$

or in decibel units

$$\mu_{\Omega_p} \,(\text{dBm})(d) = \mu_{\Omega_p} \,(\text{dBm})(d_o) - 10\beta \log_{10} \left( \frac{d}{d_o} \right) - 10n \log_{10} \left( \frac{f}{f_o} \right) + 10\log_{10} \alpha_0,$$

where $d$ is in units of kilometers and $d_o = 1.6 \text{ km}$.

• The parameter $\alpha_0$ is a correction factor used to account for different BS and MS antenna heights, transmit powers, and antenna gains.
Lee’s Area-to-area Model

• The following set of *nominal* conditions are assumed in Lee’s area-to-area model:
  
  – frequency $f_o = 900$ MHz  
  – BS antenna height = 30.48 m  
  – BS transmit power = 10 watts  
  – BS antenna gain = 6 dB above dipole gain  
  – MS antenna height = 3 m  
  – MS antenna gain = 0 dB above dipole gain

• If the actual conditions are different from those listed above, then we compute the following parameters:

  \[
  \begin{align*}
  \alpha_1 &= \left( \frac{\text{BS antenna height (m)}}{30.48 \text{ m}} \right)^2 \\
  \alpha_2 &= \left( \frac{\text{MS antenna height (m)}}{3 \text{ m}} \right)^\kappa \\
  \alpha_3 &= \frac{\text{transmitter power}}{10 \text{ watts}} \\
  \alpha_4 &= \frac{\text{BS antenna gain with respect to } \lambda_c/2 \text{ dipole}}{4} \\
  \alpha_5 &= \text{different antenna-gain correction factor at the MS}
  \end{align*}
  \]
Lee’s Area-to-area Model

- The parameters $\beta$ and $\mu_{\Omega_p}(d_o)$ have been found from empirical measurements, and are listed in the Table below.

<table>
<thead>
<tr>
<th>Terrain</th>
<th>$\mu_{\Omega_p}(d_o)$ (dBm)</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Space</td>
<td>-45</td>
<td>2</td>
</tr>
<tr>
<td>Open Area</td>
<td>-49</td>
<td>4.35</td>
</tr>
<tr>
<td>North American Suburban</td>
<td>-61.7</td>
<td>3.84</td>
</tr>
<tr>
<td>North American Urban (Philadelphia)</td>
<td>-70</td>
<td>3.68</td>
</tr>
<tr>
<td>North American Urban (Newark)</td>
<td>-64</td>
<td>4.31</td>
</tr>
<tr>
<td>Japanese Urban (Tokyo)</td>
<td>-84</td>
<td>3.05</td>
</tr>
</tbody>
</table>

- For $f_c < 450$ MHz in a suburban or open area, $n = 2$ is recommended. In an urban area with $f_c > 450$ MHz, $n = 3$ is recommended.

- The value of $\kappa$ in is also determined from empirical data as

$$
\kappa = \begin{cases} 
2 & \text{for a MS antenna height } > 10 \text{ m} \\
3 & \text{for a MS antenna height } < 3 \text{ m}
\end{cases}.
$$
Lee’s Area-to-area Model

- The path loss $L_p$ (dB) is the difference between the transmitted and received field strengths, $L_p$ (dB) = $\mu_{\Omega_p}$(dBm)($d$) − $\mu_{\Omega_t}$(dBm).

- To compare directly with the Okumura-Hata model, we assume an isotropic BS antenna with 0 dB gain, so that $\alpha_4 = -6$ dB.

- Then by using the same parameters as before, $h_b = 70$ m, $h_m = 1.5$m, $f_c = 900$ MHz, a nominal BS transmitter power of 40 dBm (10 watts), and the parameters in the Table for $\mu_{\Omega_p}$(dBm)($d_o$) and $\beta$, the following path losses are obtained:

$$L_p$ (dB) = \begin{cases} 
85.74 + 20.0 \log_{10} d & \text{Free Space} \\
84.94 + 43.5 \log_{10} d & \text{Open Area} \\
98.68 + 38.4 \log_{10} d & \text{Suburban} \\
107.31 + 36.8 \log_{10} d & \text{Philadelphia} \\
100.02 + 43.1 \log_{10} d & \text{Newark} \\
122.59 + 30.5 \log_{10} d & \text{Tokyo} 
\end{cases}$$
Path loss obtained by using Lee’s method; $h_b = 70$ m, $h_m = 1.5$ m, $f_c = 900$ Mhz, and an isotropic BS antenna.
COST231-Model

- COST231 models are for propagation in the PCS band.
- Path losses experienced at 1845 MHz are about 10 dB larger than those experienced at 955 MHz.
- The COST-231 Hata model for NLOS propagation is

\[ L_p = A + B \log_{10}(d) + C \]

where

\[ A = 46.3 + 33.9 \log_{10}(f_c) - 13.82 \log_{10}(h_b) - a(h_m) \]
\[ B = 44.9 - 6.55 \log_{10}(h_b) \]
\[ C = \begin{cases} 
0 & \text{medium city and suburban areas with moderate tree density} \\
3 & \text{for metropolitan centers} 
\end{cases} \]
COST231-Walfish-Ikegami LOS Model

• For LOS propagation in a street canyon, the path loss is

\[ L_p = 42.6 + 26 \log_{10}(d) + 20 \log_{10}(f_c), \quad d \geq 20 \, m \]

where the first constant is chosen so that \( L_p \) is equal to the free-space path loss at a distance of 20 m.

• The model parameters are the distance \( d \) (km) and carrier frequency \( f_c \) (MHz).
Definition of parameters used in the COST231-Walfish-Ikegami model.
• For NLOS propagation, the path loss is composed of three terms, viz.,

\[ L_p = \begin{cases} 
L_o + L_{rts} + L_{msd} & \text{for } L_{rts} + L_{msd} \geq 0 \\
L_o & \text{for } L_{rts} + L_{msd} < 0
\end{cases} \]

- The free-space loss is

\[ L_o = 32.4 + 20\log_{10}(d) + 20\log_{10}(f_c) \]

- The roof-top-to-street diffraction and scatter loss is

\[ L_{rts} = -16.9 - 10\log_{10}(w) + 10\log_{10}(f_c) + 20\log_{10}\Delta h_m + L_{ori} \]

where

\[ L_{ori} = \begin{cases} 
-10 + 0.354(\phi) , & 0 \leq \phi \leq 35^o \\
2.5 + 0.075(\phi - 35) , & 35 \leq \phi \leq 55^o \\
4.0 - 0.114(\phi - 55) , & 55 \leq \phi \leq 90^o
\end{cases} \]

\[ \Delta h_m = h_{Roof} - h_m \]
- The multi-screen diffraction loss is

\[ L_{\text{msd}} = L_{\text{bsh}} + k_a + k_d \log_{10}(d) + k_f \log_{10}(f_c) - 9 \log_{10}(b) \]

where

\[ L_{\text{bsh}} = \begin{cases} 
-18 \log_{10}(1 + \Delta h_b) & \text{if } h_b > h_{\text{Roof}} \\
0 & \text{if } h_b \leq h_{\text{Roof}} 
\end{cases} \]

\[ k_a = \begin{cases} 
54 & \text{if } h_b > h_{\text{Roof}} \\
54 - 0.8 \Delta h_b & \text{if } d \geq 0.5 \text{km and } h_b \leq h_{\text{Roof}} \\
54 - 0.8 \Delta h_b d/0.5 & \text{if } d < 0.5 \text{km and } h_b \leq h_{\text{Roof}} 
\end{cases} \]

\[ k_d = \begin{cases} 
18 & \text{if } h_b > h_{\text{Roof}} \\
18 - 15 \Delta h_b / h_{\text{Roof}} & \text{if } h_b \leq h_{\text{Roof}} 
\end{cases} \]

\[ k_f = -4 + \begin{cases} 
0.7(f_c/925 - 1) & \text{medium city and suburban} \\
1.5(f_c/925 - 1) & \text{metropolitan area} 
\end{cases} \]

and

\[ \Delta h_b = h_b - h_{\text{Roof}} \]
• $k_a$ is the increase in path loss for BS antennas below the roof tops of adjacent buildings.

• $k_d$ and $k_f$ control the dependency of the multi-screen diffraction loss on the distance and frequency, respectively.

• The model is valid for the following ranges of parameters, $800 \leq f_c \leq 2000$ (MHz), $4 \leq h_b \leq 50$ (m), $1 \leq h_m \leq 3$ (m), and $0.02 \leq d \leq 5$ (km).

• The following default values are recommended, $b = 20 \ldots 50$ (m), $w = b/2$, $\phi = 90^\circ$, and $h_{\text{Roof}} = 3 \times \text{number of floors} + \text{roof}$ (m), where $\text{roof} = 3$ (m) pitched and 0 (m) flat.
Shadowing

- Shadows are very often modeled as being log-normally distributed.
- Let

\[ \Omega_v = E[\alpha(t)], \quad \mu_{\Omega_v} = E[\Omega_v] \]
\[ \Omega_p = E[\alpha^2(t)], \quad \mu_{\Omega_p} = E[\Omega_p] \]

- Then distributions of \( \Omega_v \) and \( \Omega_p \) are

\[
\begin{align*}
p_{\Omega_v}(x) &= \frac{2\xi}{x\sigma_{\Omega}\sqrt{2\pi}} \exp \left\{ -\frac{(10\log_{10}x^2 - \mu_{\Omega_v \text{ (dBm)}})^2}{2\sigma_{\Omega}^2} \right\} \\
p_{\Omega_p}(x) &= \frac{\xi}{x\sigma_{\Omega}\sqrt{2\pi}} \exp \left\{ -\frac{(10\log_{10}x - \mu_{\Omega_p \text{ (dBm)}})^2}{2\sigma_{\Omega}^2} \right\}
\end{align*}
\]

where

\[
\begin{align*}
\mu_{\Omega_v \text{ (dBm)}} &= 30 + 10E[\log_{10}\Omega_v^2] \\
\mu_{\Omega_p \text{ (dBm)}} &= 30 + 10E[\log_{10}\Omega_p]
\end{align*}
\]

and \( \xi = \ln 10/10 \).
Shadowing

- By using a transformation of random variables, $\Omega_v \text{ (dBm)} = 30 + 10\log_{10}\Omega_v^2$ and $\Omega_p \text{ (dBm)} = 30 + 10\log_{10}\Omega_p$ have the Gaussian densities

$$p_{\Omega_v \text{ (dBm)}}(x) = \frac{1}{\sqrt{2\pi}\sigma_\Omega} \exp\left\{ -\frac{(x - \mu_{\Omega_v \text{ (dBm)}})^2}{2\sigma_\Omega^2} \right\}$$

$$p_{\Omega_p \text{ (dBm)}}(x) = \frac{1}{\sqrt{2\pi}\sigma_\Omega} \exp\left\{ -\frac{(x - \mu_{\Omega_p \text{ (dBm)}})^2}{2\sigma_\Omega^2} \right\} .$$

- Note that the standard deviation $\sigma_\Omega$ of $\Omega_v \text{ (dBm)}$ and $\Omega_p \text{ (dBm)}$ are the same. However, for Rician fading channels the means differ by

$$\mu_{\Omega_p \text{ (dBm)}} = \mu_{\Omega_v \text{ (dBm)}} + 10 \cdot \log_{10}C(K)$$

where

$$C(K) = \frac{4e^{2K}(K + 1)}{\pi_1 F_1^2(3/2, 1; K)}$$

$$_1 F_1(\cdot; \cdot; \cdot)$ is the confluent hypergeometric function (see Chap. 2, Appendix 3).

- Note that $C(0) = 4/\pi$, $C(\infty) = 1$, and $1 \leq C(K) \leq 4/\pi$ for $0 \leq K \leq \infty$.  

Shadow Simulation

- Shadows can be modelled by low-pass filtering white noise.
  - Here we suggest a first-order low pass digital filter.

- In a discrete-time simulation, the local mean $\Omega_{k+1} \text{ (dBm)}$ at step $k+1$ is generated recursively as follows:

$$
\Omega_{k+1} \text{ (dBm)} = \xi \Omega_k \text{ (dBm)} + (1 - \xi) v_k
$$

  - $k$ is the step index.
  - $\{v_k\}$ is a sequence of independent zero-mean Gaussian random variables with variance $\tilde{\sigma}^2$.
  - $\xi$ controls the shadow correlation

- The autocorrelation function of $\Omega_k \text{ (dBm)}$ can be derived as:

$$
\phi_{\Omega_{(dBm)\Omega_{(dBm)}}}(n) = \frac{1 - \xi}{1 + \xi \tilde{\sigma}^2 |n|}
$$
• The variance of log-normal shadowing is

\[ \sigma^2 = \phi_{\Omega(dBm)\Omega(dBm)}(0) = \frac{1 - \xi}{1 + \xi} = \frac{1 - \tilde{\sigma}^2}{1 + \tilde{\sigma}^2} \]

Consequently, we can express the autocorrelation of \( \Omega_k \) as

\[ \phi_{\Omega(dBm)\Omega(dBm)}(n) = \sigma^2 \xi^{|n|} \]

Notice that the shadows decorrelated exponentially with the time lag in the autocorrelation function.

• Suppose we use discrete-time simulation, where each simulation step corresponds to \( T \) seconds.

  – For a mobile station traveling at velocity \( v \), the distance traveled in \( T \) seconds is \( vT \) meters.
  – Let \( \xi_D \) be the shadow correlation between two points separated by a spatial distance of \( D \) meters.
  – Then the time autocorrelation of the shadowing is

\[ \phi_{\Omega(dBm)\Omega(dBm)}(n) \equiv \phi_{\Omega(dBm)\Omega(dBm)}(nT) = \sigma^2 \xi\left(\frac{vT}{D}\right)^{|n|} \]

Measurements in Stockholm have shown \( \xi_D = 0.1 \) for \( D = 30 \) meters (roughly). However, this can vary greatly depending on local topography.
Co-channel interference on the forward channel

The mobile station is being served by the center base station.
• At a particular location, let \( d = (d_0, d_1, \ldots, d_{N_I}) \) be the vector of distances between a mobile station and the serving base station BS\(_0\) and \( N_I \) co-channel base stations BS\(_k\), \( k = 1, \ldots, N_I \).

• The received signal power \( \Omega_p(d) \), is a Gaussian random variable that depends on the distance \( d \) through the path loss model, i.e.,

\[
\mu_{\Omega_p(d)} = E[\Omega_p(d)] = \mu_{\Omega_p(d_o)} - 10\beta \log_{10}(d/d_o)
\]

• Experiments have verified that co-channel interferers add noncoherently (power addition) rather than coherently (amplitude addition).

• The C/I a function of the vector \( d \) is

\[
\Lambda(d) = \frac{\Omega_p(d_0)}{\sum_{k=1}^{N_I} \Omega_p(d_k)}
\]

or in decibel units

\[
\Lambda(d)(dB) = \Omega_p(d_0) - 10 \log_{10} \left( \sum_{k=1}^{N_I} \Omega_p(d_k) \right)
\]

• The outage probability given vector \( d \) is

\[
O(d) = P_r \left( \Lambda(d)(dB) < \Lambda_{th}(dB) \right)
\]

• Although the \( \Omega_p(d_k) \) are log-normal random variables, the sum \( \sum_{k=1}^{N_I} \Omega_p(d_k) \) is not a log-normal random variable.
Multiple Log-normal Interferers

- Consider the sum of $N_I$ log-normal random variables

$$I = \sum_{k=1}^{N_I} \Omega_k = \sum_{k=1}^{N_I} 10^{\Omega_k(dBm)/10}$$

where the $\Omega_k$ (dBm) are independent Gaussian random variables with mean $\mu_{\Omega_k}$ (dBm) and variance $\sigma_{\Omega_k}^2$.

- The sum $I$ is commonly approximated by another log-normal random variable with appropriately chosen parameters, i.e.,

$$I = \sum_{k=1}^{N_I} 10^{\Omega_k(dBm)/10} \approx 10^{Z(dBm)/10} = \hat{I}$$

where $Z_{(dBm)}$ is a Gaussian random variable with mean $\mu_Z$ (dBm) and variance $\sigma_Z^2$.

- The task is to find $\mu_Z$ (dBm) and $\sigma_Z^2$. 

Fenton-Wilkinson Method

- The mean $\mu_Z \text{ (dBM)}$ and variance $\sigma^2_Z$ of $Z_{\text{(dBM)}}$ are obtained by matching the first two moments of $I$ and $\hat{I}$.

- Switching from base 10 to base $e$:

$$\Omega_k = 10^{\Omega_k \text{ (dBm)}/10} = e^{\xi \Omega_k \text{ (dBm)}} = e^{\hat{\Omega}_k}$$

where $\hat{\Omega}_k = \xi \Omega_k \text{ (dBm)}$ and $\xi = (\ln 10)/10 = 0.23026$.

- Note that

$$\mu_{\hat{\Omega}_k} = \xi \mu_{\Omega_k} \text{ (dBm)}$$

$$\sigma^2_{\hat{\Omega}_k} = \xi^2 \sigma^2_{\Omega_k}$$

- The $n$th moment of the log-normal random variable $\Omega_k$ can be obtained from the moment generating function of the Gaussian random variable $\hat{\Omega}_k$ as

$$\mathbb{E}[\Omega_k^n] = \mathbb{E}[e^{n\hat{\Omega}_k}] = e^{n\mu_{\hat{\Omega}_k} + (1/2)n^2\sigma^2_{\hat{\Omega}}}$$

- Here we have assumed identical shadow variances, $\sigma^2_{\hat{\Omega}_k} = \sigma^2_{\hat{\Omega}}$, which is a reasonable assumption.
• Suppose that $\hat{\Omega}_1, \ldots, \hat{\Omega}_{N_I}$ are independent with means $\mu_{\hat{\Omega}_1}, \ldots, \mu_{\hat{\Omega}_{N_I}}$ and identical variances $\sigma^2_{\hat{\Omega}}$.

• The appropriate moments of the log-normal approximation are obtained by equating the means on both sides of

$$\mu_I = E[I] = \sum_{k=1}^{N_I} E[e^{\hat{\Omega}_k}] \approx E[e^{\hat{Z}}] = E[\hat{I}] = \mu_{\hat{I}}$$

where $\hat{Z} = \xi Z_{(dBm)}$.

• This gives

$$\left( \sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}} \right) e^{(1/2)\sigma^2_{\hat{\Omega}}} = e^{\mu_{\hat{Z}} + (1/2)\sigma^2_{\hat{Z}}} \quad (1)$$

• Also equate the variances on both sides of

$$\sigma^2_I = E[I^2] - \mu_I^2 \approx E[\hat{I}^2] - \mu_{\hat{I}}^2 = \sigma^2_{\hat{I}}$$

• This gives

$$\left( \sum_{k=1}^{N_I} e^{2\mu_{\hat{\Omega}_k}} \right) e^{\sigma^2_{\hat{\Omega}} (e^{\sigma^2_{\hat{\Omega}}} - 1)} = e^{2\mu_{\hat{Z}} e^{\sigma^2_{\hat{Z}}} (e^{\sigma^2_{\hat{Z}}} - 1)} \quad (2)$$
• To obtain $\mu_{\hat{Z}}$ and $\sigma^2_{\hat{Z}}$

1. Square Eq. (1) and divide by Eq. (2) to obtain $\sigma^2_{\hat{Z}}$.
2. Obtain $\mu_{\hat{Z}}$ from Eq. (1)

• The above procedure yields

\[
\sigma^2_{\hat{Z}} = \ln \left( \left( e^{\sigma^2_{\hat{\Omega}}} - 1 \right) \frac{\sum_{k=1}^{N_I} e^{2\mu_{\hat{\Omega}_k}}}{\left( \sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}} \right)^2} + 1 \right)
\]

\[
\mu_{\hat{Z}} = \frac{\sigma^2_{\hat{\Omega}} - \sigma^2_{\hat{Z}}}{2} + \ln \left( \sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}} \right)
\]

• Given the means $\mu_{\hat{\Omega}_1}, \ldots, \mu_{\hat{\Omega}_{NI}}$ and variance $\sigma^2_{\hat{\Omega}}$, $\mu_{\hat{Z}}$ and $\sigma^2_{\hat{Z}}$ are easily obtained.

• Finally, we convert back to base 10 by scaling, such that

\[
\mu_{Z \text{ (dBm)}} = \xi^{-1} \mu_{\hat{Z}}
\]

\[
\sigma^2_{Z} = \xi^{-2} \sigma^2_{\hat{Z}}
\]

where $\xi = 0.23026$. 


• Fenton’s method breaks down in the prediction of the first and second moments for $\sigma_\Omega > 4$ dB.
  – Schwartz and Yeh’s method yields the exact first and second moments.

• However, Fenton’s method accurately predicts the tails of the complementary distribution function $cdfs F_c^I(x) = P_r(I \geq x)$ and the cdf $F_I(x) = 1 - F_c^I(x) = P_r(I < x)$.
  – We are interested in the accuracy of the approximations

$$F_I(x) \approx Q\left(\frac{\ln x - \mu^z}{\sigma^{z}}\right)$$

$$F_c^I(x) \approx 1 - Q\left(\frac{\ln x - \mu^z}{\sigma^{z}}\right)$$

when $x$ is small and large, respectively.
  – The cdf is more important than the cdf for outage calculations and predictions, since outages typically occur when the interference is large.
Comparison of the cdf for the sum of two and six log-normal random variables with various approximations; $\sigma_\Omega = 6$ dB.
Comparison of the cdfc for the sum of two log-normal random variables with various approximations; $\sigma_\Omega = 6$ dB.
Comparison of the cdf for the sum of six log-normal random variables with various approximations; $\sigma_\Omega = 6$ dB.
Comparison of the cdfc for the sum of six log-normal random variables with various approximations; $\sigma_\Omega = 12$ dB.
Outage with Multiple Interferers

1. First obtain the mean and variance

\[ \mu_Z = \mu_{\hat{Z}} / \xi \]
\[ \sigma^2_Z = \sigma^2_{\hat{Z}} / \xi^2 \]
\[ \xi = 0.23026 \]

2. Treat the average CIR as Gaussian distributed with mean and variance

\[ \mu_{\Lambda(d)} = \mu_{\Omega(d_0)} - \mu_Z \text{ (dBm)} \]
\[ \sigma^2_{\Lambda(d)} = \sigma^2_{\Omega} + \sigma^2_{Z} . \]

3. Compute the outage for a given location, described by \( d \)

\[ O(d) = Q \left( \frac{\mu_{\Omega(d_0)} - \mu_Z - \Lambda_{\text{th}}(\text{dB})}{\sqrt{\sigma^2_{\Omega} + \sigma^2_{Z}}} \right) \]

4. Average over all locations \( d \) by Monte Carlo integration

\[ O = \int_{\mathbb{R}^N} O(d) p_d(d) dd \]
Single Co-channel Interferer

- For a single co-channel interferer

\[
p_{\Lambda(d)_{(dB)}}(x) = \frac{1}{\sqrt{4\pi\sigma_\Omega}} \exp\left\{ -\frac{(x - \mu_{\Lambda(d)_{(dB)}})^2}{4\sigma_\Omega^2} \right\}
\]

where

\[
\mu_{\Lambda(d)_{(dB)}} = \mu_{\Omega(d_0)_{(dB)}} - \mu_{\Omega(d_1)_{(dB)}}
\]

- The outage for a given d is

\[
O(d) = \Pr(\Lambda(d)_{(dB)} < \Lambda_{th(dB)})
\]

\[
= \int_{-\infty}^{\Lambda_{th(dB)}} \frac{1}{\sqrt{4\pi\sigma_\Omega}} \exp\left\{ -\frac{(x - \mu_{\Lambda(d)_{(dB)}})^2}{4\sigma_\Omega^2} \right\} \, dx
\]

\[
= Q\left( \frac{\mu_{\Lambda(d)_{(dB)}} - \Lambda_{th(dB)}}{\sqrt{2\sigma_\Omega}} \right)
\]
Worst case interference from a single co-channel base-station.

- In this case $d = (R, D - R)$.
- The worst case outage due to a single co-channel interferer is

$$O(R) = Q\left(\frac{\mu\Omega(R)_{(dB)} - \mu\Omega(D-R)_{(dB)} - \Lambda_{th} (dB)}{\sqrt{2}\sigma\Omega}\right)$$
Using a simple inverse-\( \beta \) path loss characteristic

\[
\mu_{\Omega_{\text{dB}}} = \Omega_{\text{dB}}(d_o) - 10\beta \log_{10}(d/d_o)
\]

gives

\[
O(R) = Q \left( \frac{10 \log_{10} \left( \frac{D}{R} - 1 \right)^\beta - \Lambda_{\text{th}} \text{(dB)}}{\sqrt{2}\sigma_{\Omega}} \right)
\]

The minimum CIR margin on the cell fringe is

\[
M_A = 10 \log_{10} \left( \frac{D}{R} - 1 \right)^\beta - \Lambda_{\text{th}} \text{(dB)}
\]

For an ideal hexagonal layout \( \frac{D}{R} = \sqrt{3}N \), so that

\[
N = \frac{1}{3} \left[ 10 \frac{M_A + \Lambda_{\text{th}} \text{(dB)}}{10\beta} + 1 \right]^2
\]

- A small cluster size is achieved by making the margin \( M_A \) and receiver threshold \( \Lambda_{\text{th}} \) small.
Rician/Multiple Rayleigh Interferers

- Sometimes propagation conditions exist such that the received signals experience fading, but not shadowing. In this section, we calculate the outage probability for the case of fading only.
  - The received signal may consist of a direct line of sight (LoS) component, or perhaps a specular component, accompanied by a diffuse component. The envelope of the received desired signal experiences Ricean fading.
  - The interfering signals are often assumed to be Rayleigh faded, because a direct LoS is unlikely to exist due to the larger physical distances between the co-channel interferers and the receiver.

- Let the instantaneous power in the desired signal and the $N_I$ interfering signals be denoted by $s_0$ and $s_k$, $k = 1, \ldots, N_I$, respectively. Note that $s_i = \alpha_i^2$, where $\alpha_i^2$ is the squared-envelope.

- The carrier-to-interference ratio is defined as $\lambda = s_0 / \sum_{k=1}^{N_I} s_k$, and for a specified receiver threshold $\lambda_{th}$, the outage probability is
  $$O_I = P (\lambda < \lambda_{th}) .$$
For the case of a single interferer, the outage probability reduces to the simple closed form

\[ O_I = \frac{\lambda_{th}}{\lambda_{th} + A_1} \exp \left\{ -\frac{K A_1}{\lambda_{th} + A_1} \right\}, \]

where \( K \) is the Rice factor of the desired signal, \( A_1 = \Omega_0/(K + 1)\Omega_1 \), and \( \Omega_k = E[s_k] \).

If the desired signal is Rayleigh faded, then the outage probability can be obtained by setting \( K = 0 \).
Multiple Interferers

- For the case of multiple interferers, each with mean power $\Omega_k$, the outage probability has the closed form

$$O_I = 1 - \sum_{k=1}^{N_I} \left[ 1 - \frac{\lambda_{th}}{\lambda_{th} + A_k} \exp \left\{ -\frac{KA_k}{\lambda_{th} + A_k} \right\} \right] \prod_{\substack{j=1 \atop j \neq k}}^{N_I} \frac{A_j}{A_j - A_k},$$

where $A_k = \Omega_0/(K + 1)\Omega_k$. This expression is only valid if $\Omega_i \neq \Omega_j$ when $i \neq j$, i.e., the different interferers have different mean power.

- If all the interferers have the same mean power, then the outage probability can be derived as

$$O_I = \frac{\lambda_{th}}{\lambda_{th} + A_1} \exp \left\{ -\frac{KA_1}{\lambda_{th} + A_1} \right\} \times \sum_{k=0}^{N_I-1} \left( \frac{A_1}{\lambda_{th} + A_1} \right)^k \sum_{m=0}^{k} \binom{k}{m} \frac{1}{m!} \left( \frac{K\lambda_{th}}{\lambda_{th} + A_1} \right)^m.$$

- If the desired signal is Rayleigh faded, then the probability of outage with multiple Rayleigh faded interferers can be obtained by setting $K = 0$. 

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Probability of outage with multiple interferers. The desired signal is Ricean faded with various Rice factors, while the interfering signals are Rayleigh faded and of equal power; $\lambda_{th} = 10.0$ dB.

\[
\Lambda = \frac{\Omega_0}{N_I\Omega_1}
\]