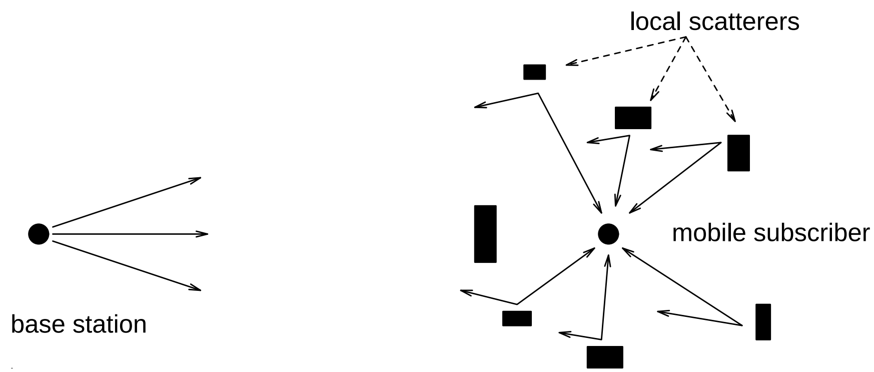
ECE6604
PERSONAL & MOBILE COMMUNICATIONS

Week 3

Flat Fading Channels
Envelope Distribution

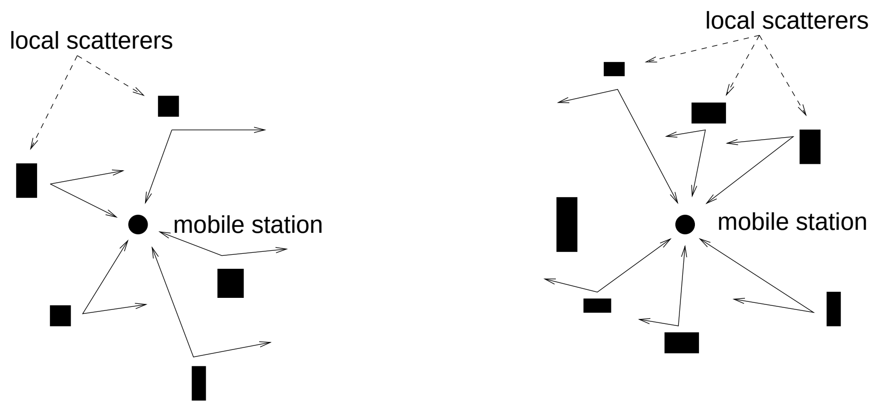
Reading: Chapter 2, 2.1,2.1.3

Multipath-Fading Mechanism

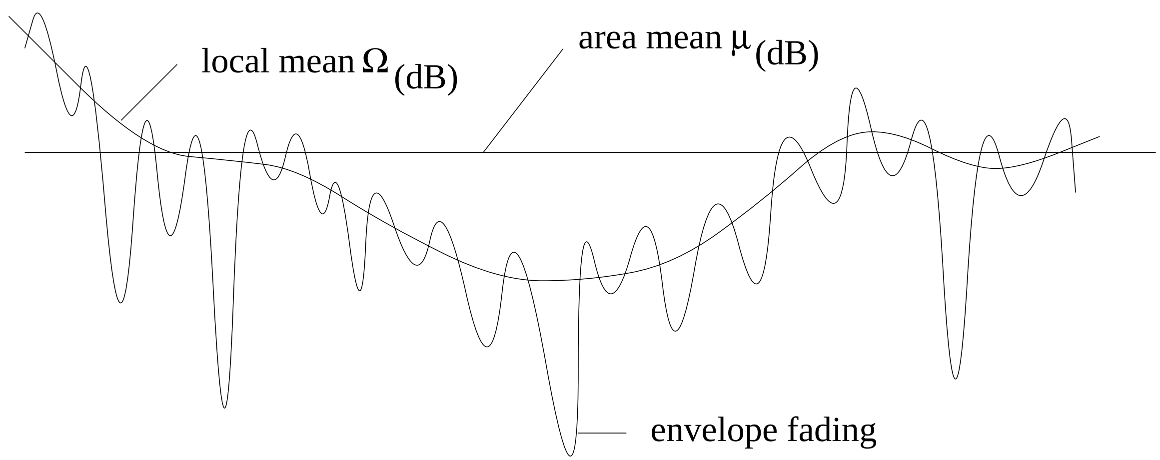


A typical macrocellular mobile radio environment.

Multipath-Fading Mechanism

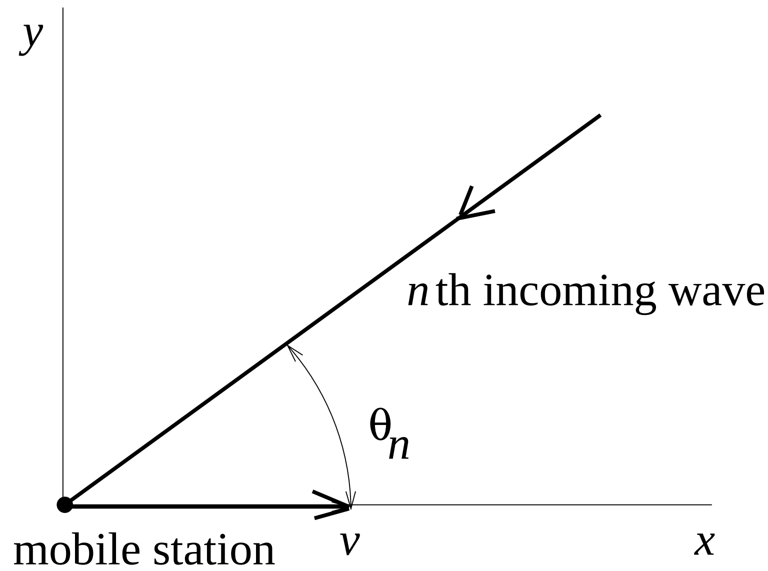


Typical mobile-to-mobile radio propagation environment.



Path loss, shadowing, envelope fading.

Doppler Shift



2-D Model of a typical wave component incident on a mobile station (MS).

- Assuming 2-D propagation, the **Doppler shift** is $f_{D,n} = f_m \cos\theta_n$, where $f_m = v/\lambda_c$ (λ_c is the carrier wavelength, v is the mobile station velocity).

Multipath Propagation

- Consider the transmission of the band-pass signal

$$s(t) = \mathrm{Re}\left\{\tilde{s}(t)e^{j2\pi f_c t}\right\}$$

- At the receiver antenna, the nth plane wave arrives at angle θ_n and experiences Doppler shift $f_{D,n} = f_m \cos\theta_n$ and propagation delay τ_n.

- If there are N propagation paths, the received bandpass signal is

$$r(t) = \mathrm{Re}\left[\sum_{n=1}^{N} C_n \mathrm{e}^{j\phi_n - j2\pi c\tau_n/\lambda_c + j2\pi(f_c + f_{D,n})t}\tilde{s}(t-\tau_n)\right] \ ,$$

  where C_n, ϕ_n, $f_{D,n}$ and τ_n are the amplitude, phase, Doppler shift and time delay, respectively, associated with the nth propagation path, and c is the speed of light.

- The delay $\tau_n = d_n/c$ is the propagation delay associated with the nth propagation path, where d_n is the length of the path. The path lengths, d_n, will depend on the physical scattering geometry which we have not specified at this point.

Multipath Propagation

- The received bandpass signal $r(t)$ has the form

$$r(t) = \text{Re}\left[\tilde{r}(t)e^{j2\pi f_c t}\right]$$

where the received complex envelope is

$$\tilde{r}(t) = \sum_{n=1}^{N} C_n e^{j\phi_n(t)} \tilde{s}(t - \tau_n)$$

and

$$\phi_n(t) = \phi_n - 2\pi c\tau_n/\lambda_c + 2\pi f_{D,n} t$$

is the time-variant phase associated with the nth propagation path.

  – Note that the nth component varies with the Doppler frequency $f_{D,n}$.
  – The term $c\tau_n/\lambda_c$ is the propagation distance $c\tau_n$ normalized by the carrier wavelength λ_c. For cellular frequencies (900 MHz), λ_c is on the order of a foot.
  – The phase ϕ_n is introduced by the nth scatterer randomly, and can be assumed to be uniformly distributed on $[-\pi, \pi)$.
  – The received phase at any time t, $\phi_n(t)$ is uniformly distributed on $[-\pi, \pi)$.

Flat Fading - impulse response

- The received waveform is given by the convolution

$$\tilde{r}(t) = \int_0^t g(t,\tau)\tilde{s}(t-\tau)\mathrm{d}\tau$$

- It follows that the channel can be modeled by a linear time-variant filter having the **time-variant impulse response**

$$g(t,\tau) = \sum_{n=1}^{N} C_n e^{j\phi_n(t)}\delta(\tau - \tau_n)$$

- If the differential path delays $\tau_i - \tau_j$ are all very small compared to the modulation symbol period, T, then the τ_n can be replaced by the mean delay μ_τ inside the delta function. Note that this approximation is not applied to the channel phases $\phi_n(t)$, since small changes in τ_n result in large changes in $\phi_n(t)$.
  - The channel impulse response has the approximate form

$$g(t,\tau) = g(t)\delta(\tau - \mu_\tau)\ , \qquad g(t) = \sum_{n=1}^{N} C_n e^{j\phi_n(t)}\ .$$

  - The received complex envelope is

$$\tilde{r}(t) = g(t)\tilde{s}(t-\mu_\tau) \tag{1}$$

  which experiences **fading** due to the time-varying complex channel gain $g(t)$.

Flat Fading - frequency domain

- By taking Fourier transforms of both sides of (1), the received complex envelope in the frequency domain is

$$\tilde{R}(f) = G(f) * \tilde{S}(f) e^{-j2\pi f \mu_\tau}$$

- Since the channel component $g(t)$ changes with time, it follows that $G(f)$ has a finite non-zero width in the frequency domain.
- Due to the convolution operation, the output spectrum $\tilde{R}(f)$ will be wider than the input spectrum $\tilde{S}(f)$. This broadening of the transmitted signal spectrum is caused by the channel time variations and is called **frequency spreading** or **Doppler spreading**.
  - If the maximum Doppler frequency f_m is much less than the signal bandwidth W_c, then the Doppler spreading will not distort $\tilde{S}(f)$.
  - Fortunately, this is often the case.

Channel Transfer Function - Flat Fading

- The **time-variant channel transfer function** is obtained by taking the Fourier transform of the time-variant channel impulse response $g(t,\tau)$ with respect to the delay variable τ, i.e.,

$$T(t,f) = g(t)e^{-j2\pi f \mu_\tau} \ .$$

- Since the magnitude response is $|T(t,f)| = |g(t)|$, all frequency components in the received signal are subject to the same time-variant amplitude gain $|g(t)|$ and phase response $\angle T(t,f) = \angle g(t) - 2\pi f \mu_\tau$.

- The received signal is said to exhibit "**flat fading**," because the magnitude of the time-variant channel transfer function $|T(t,f)|$ is constant (or flat) with respect to frequency variable f.

- The phase response $\angle T(t,f) = \angle g(t) - 2\pi f \mu_\tau$ is linear in f meaning that the channel delays the input signal, and gives it a time-varying attenuation and phase rotation.

Invoking the Central Limit Theorem

- Consider the transmission of an unmodulated carrier, $\tilde{s}(t) = 1$.
- For flat fading channels, the received band-pass signal has the quadrature representation

$$r(t) = g_I(t) \cos 2\pi f_c t \; - \; g_Q(t) \sin 2\pi f_c t$$

where

$$\begin{aligned} g_I(t) &= \sum_{n=1}^{N} C_n \cos \phi_n(t) \\ g_Q(t) &= \sum_{n=1}^{N} C_n \sin \phi_n(t) \end{aligned}$$

and where $\phi_n(t) = \phi_n - 2\pi c \tau_n / \lambda_c + 2\pi f_{D,n} t$.

- The phases ϕ_n are independent and uniform on the interval $[-\pi, \pi)$, and the path delays τ_n are all independent with $f_c \tau_n \gg 1$. Therefore, the phases $\phi_n(t)$ at any time t can be treated as being independent and uniformly distributed on the interval $[-\pi, \pi)$.
- In the limit $N \to \infty$, the **central limit theorem** can be invoked and $g_I(t)$ and $g_Q(t)$ can be treated as "**Gaussian random processes**," i.e., at any time t, $g_I(t)$ and $g_Q(t)$ are Gaussian random variables.
- The "**complex faded envelope**" is

$$g(t) = g_I(t) + j g_Q(t)$$

Rayleigh Fading

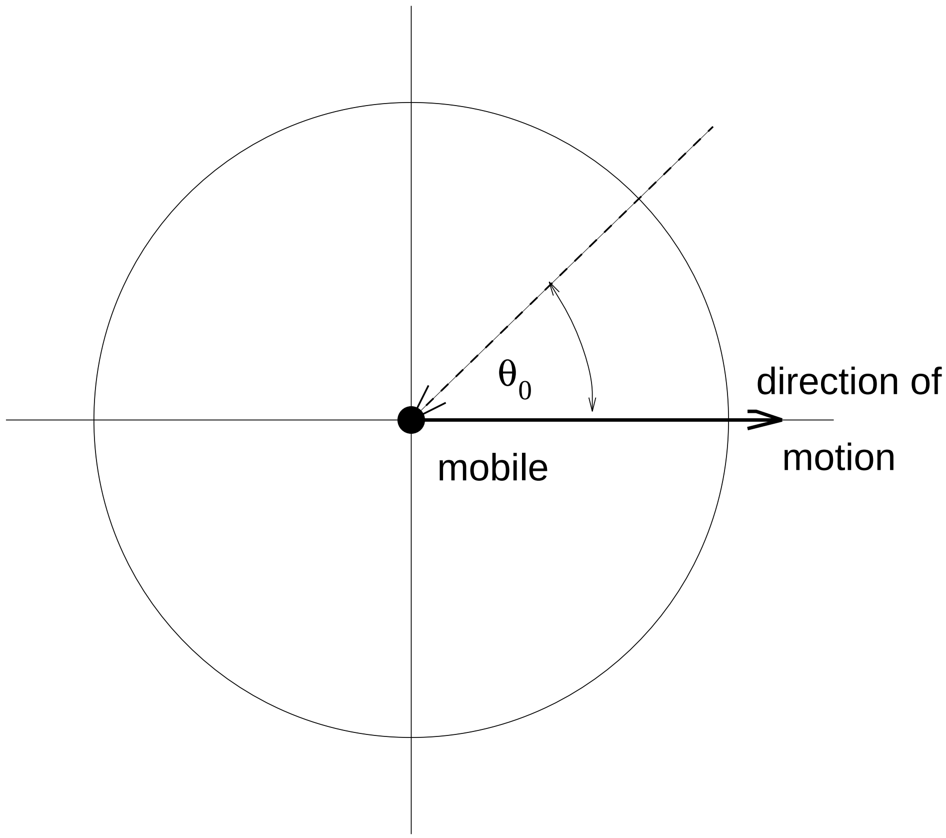
- For some types of scattering environments, $g_I(t)$ and $g_Q(t)$ at any time t_1 are independent identically distributed Gaussian random variables with zero mean and identical variance $b_0 = \mathrm{E}[g_I^2(t_1)] = \mathrm{E}[g_Q^2(t_1)]$. This typically occurs in a rich scattering environment where there is no line-of-sight or strong specular component in the received signal (i.e., there is no dominant C_n) and isotropic antennas are used. Under such conditions, the channel exhibits **Rayleigh fading**.
- The probability density function the **envelope** $\alpha = |g(t_1)| = \sqrt{g_I^2(t_1) + g_Q^2(t_1)}$ can be obtained by using a bi-variate transformation of random variables (see Appendix in textbook).
- The envelope $\alpha = |g(t_1)| = \sqrt{g_I^2(t_1) + g_Q^2(t_1)}$ is **Rayleigh** distributed at any time t_1, i.e.,

$$p_\alpha(x) = \frac{x}{b_0} \exp\left\{ -\frac{x^2}{2b_0} \right\} = \frac{2x}{\Omega_p} \exp\left\{ -\frac{x^2}{\Omega_p} \right\} , \quad x \geq 0 ,$$

  where $\Omega_p = \mathrm{E}[\alpha^2] = \mathrm{E}[g_I^2(t_1)] + \mathrm{E}[g_Q^2(t_1)] = 2b_0$ is the **average envelope power**.
- The squared-envelope α^2 at any time t_1 has the exponential distribution

$$p_{\alpha^2}(x) = \frac{1}{\Omega_p} \exp\left\{ -\frac{x}{\Omega_p} \right\} , \quad x \geq 0 .$$

Ricean Fading



A line-of-sight (LoS) or specular (strong reflected) component arrives at angle θ_0.

- For scattering environments that have a specular or LoS component, $g_I(t)$ and $g_Q(t)$ are Gaussian random processes with non-zero means $m_I(t)$ and $m_Q(t)$, respectively.
- If we again assume that $g_I(t_1)$ and $g_Q(t_1)$ at any time t_1 are independent random variables with variance $b_0 = \mathrm{E}[(g_I(t_1) - m_I(t_1))^2] = \mathrm{E}[(g_Q(t_1) - m_Q(t_1))^2]$, then the magnitude of the envelope $\alpha = |g(t_1)|$ at any time t_1 has a Rice distribution.
- With Aulin's Ricean fading model

$$\begin{aligned} m_I(t) &= \mathrm{E}[g_I(t)] = s \cdot \cos(2\pi f_m \cos(\theta_0)t + \phi_0) \\ m_Q(t) &= \mathrm{E}[g_Q(t)] = s \cdot \sin(2\pi f_m \cos(\theta_0)t + \phi_0) \end{aligned}$$

  where $f_m \cos(\theta_0)$ and ϕ_0 are the Doppler shift and random phase offset associated with the LoS or specular component, respectively.
- The envelope $\alpha(t) = |g(t)| = \sqrt{g_I^2(t) + g_Q^2(t)}$ has the **Rice** distribution

$$p_\alpha(x) = \frac{x}{b_0} \exp\left\{-\tfrac{x^2+s^2}{2b_0}\right\} I_o\left(\frac{xs}{b_0}\right) \;, \quad x \geq 0$$

  - $s^2 = m_I(t)^2 + m_Q^2(t)$ is the specular power.
  - $2b_0$ is the scatter power.
  - The **Rice factor**, $K = s^2/2b_0$, is the ratio of the power in the specular and scatter components.

- The average envelope power is $\mathrm{E}[\alpha^2] = \Omega_p = s^2 + 2b_0$ and

$$s^2 = \frac{K\Omega_p}{K+1}, \qquad 2b_0 = \frac{\Omega_p}{K+1}$$

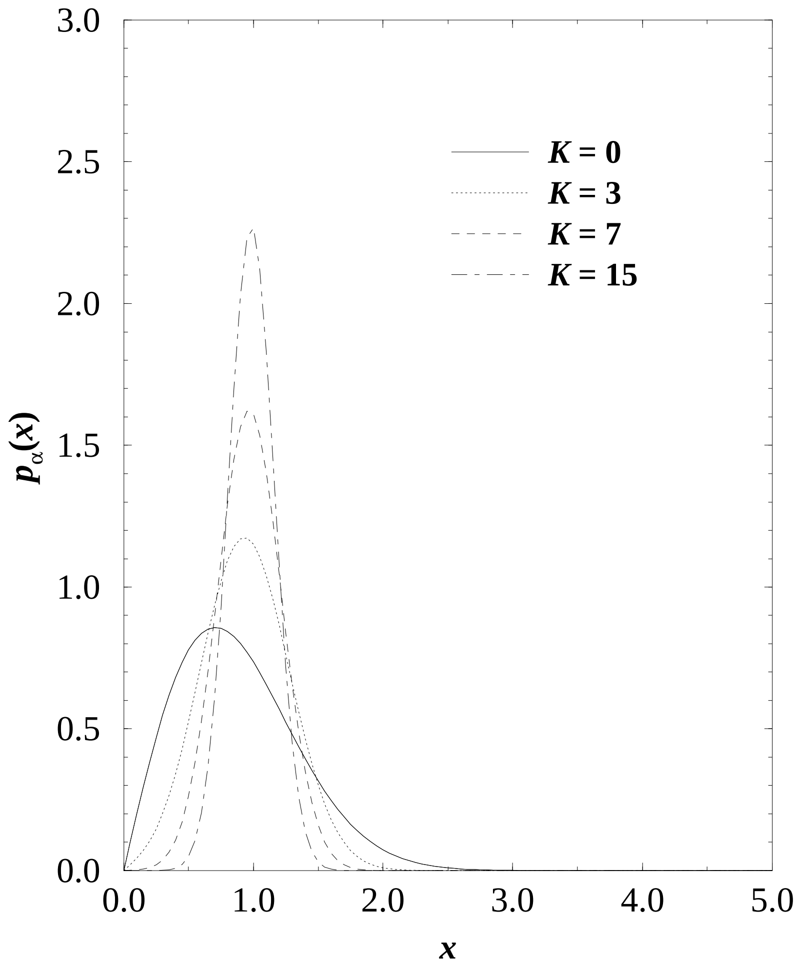
  Hence,

$$p_\alpha(x) = \frac{2x(K+1)}{\Omega_p} \exp\left\{-K - \frac{(K+1)x^2}{\Omega_p}\right\} I_o\left(2x\sqrt{\frac{K(K+1)}{\Omega_p}}\right) \ , \quad x \geq 0$$

- The squared-envelope $\alpha^2(t)$ has **non-central chi-square distribution** with two degrees of freedom

$$p_{\alpha^2}(x) = \frac{(K+1)}{\Omega_p} \exp\left\{-K - \frac{(K+1)x}{\Omega_p}\right\} I_o\left(2\sqrt{\frac{K(K+1)x}{\Omega_p}}\right) \ , \quad x \geq 0$$

- The squared-envelope is important for the performance analysis of digital communication systems because it is proportional to the received signal power and, hence, the received signal-to-noise ratio.



The Rice distribution for several values of K with $\Omega_p = 1$.

Nakagami Fading

- Nakagami fading describes the magnitude of the received complex envelope by the distribution

$$p_\alpha(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)\Omega_p^m} \exp\left\{-\frac{mx^2}{\Omega_p}\right\} \quad m \geq \frac{1}{2}$$

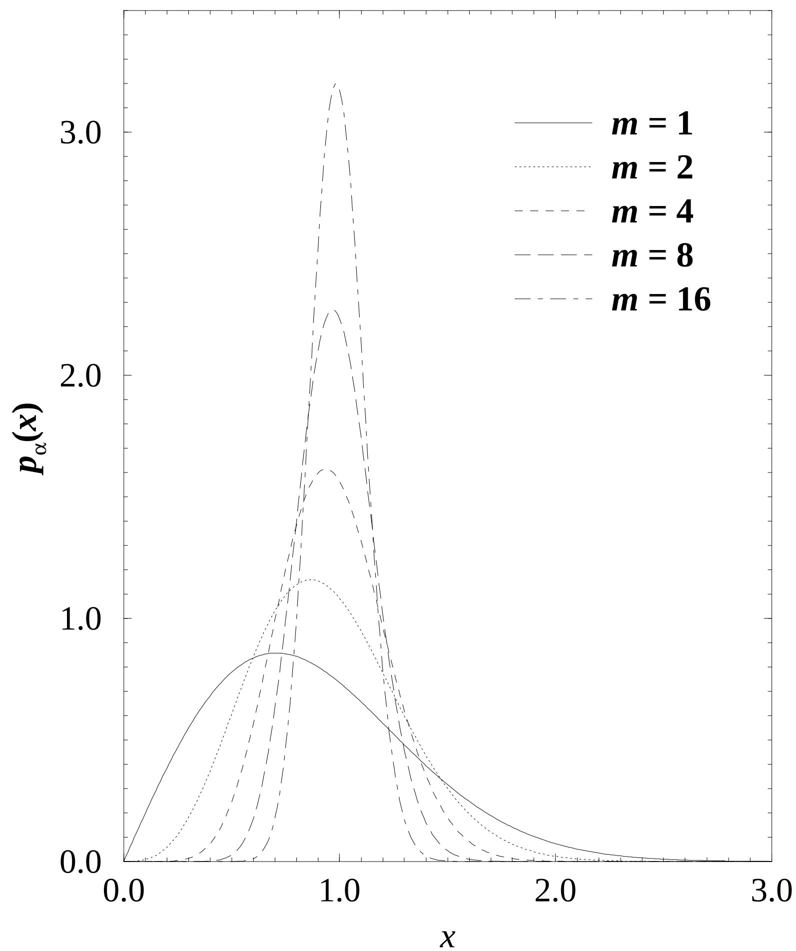
- When $m = 1$, the Nakagami distribution becomes the Rayleigh distribution, when $m = 1/2$ it becomes a one-sided Gaussian distribution, and when $m \to \infty$ the distribution approaches an impulse (no fading).

- The Rice distribution can be closely approximated with a Nakagami distribution by using the following relation between the Rice factor K and the Nakagami shape factor m

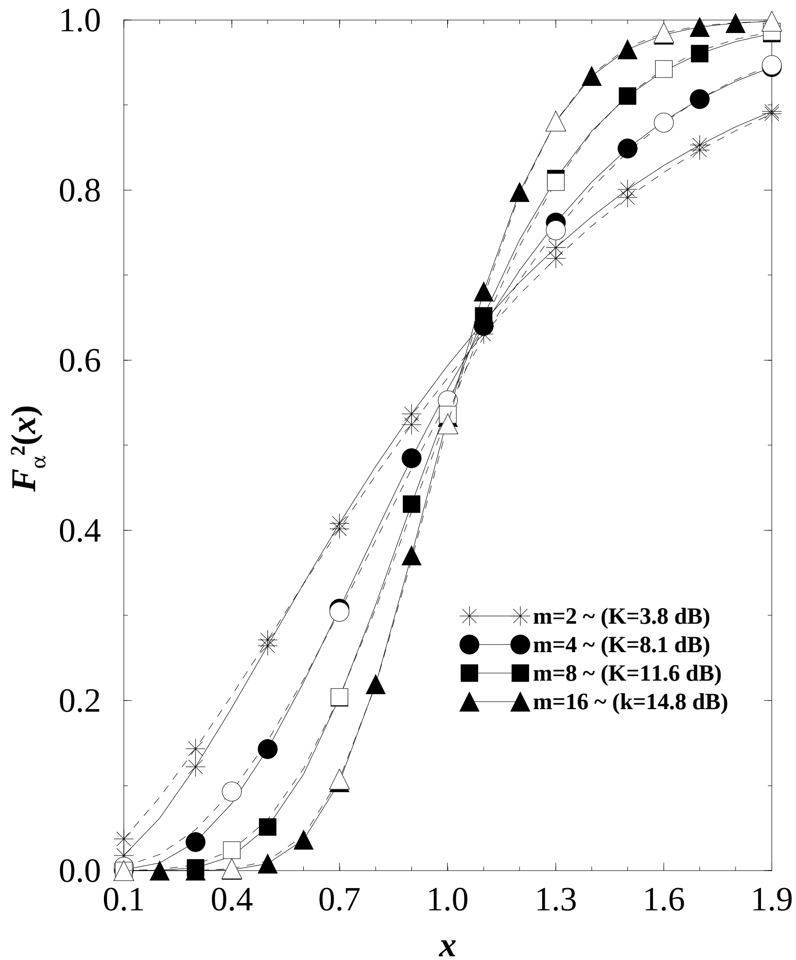
$$\begin{aligned} K &\approx \sqrt{m^2 - m} + m - 1 \\ m &\approx \frac{(K+1)^2}{(2K+1)} \ . \end{aligned}$$

- The squared-envelope has the Gamma distribution

$$p_{\alpha^2}(x) = \left(\frac{m}{2\Omega_p}\right)^m \frac{x^{m-1}}{\Gamma(m)} \exp\left\{-\frac{mx}{2\Omega_p}\right\} \quad .$$



The Nakagami pdf for several values of m *with* $\Omega_p = 1$.



Comparison of the cdf of the squared-envelope with Ricean and Nakagami fading.