Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Final Exam

Spring 2010

Tuesday May 6, 11:30am - 2:20pm

• Attempt all questions.
• All questions are of equal value.
• Open book, open notes, exam.
1a) 5 marks: The LCR at the normalized threshold $\rho$ for a 2-D isotropic scattering channel can be expressed as

$$L_R = \sqrt{2\pi f_m \rho e^{-\rho^2}},$$

where

$$\rho = \frac{R}{\sqrt{\Omega_p}} = \frac{R}{R_{\text{rms}}}$$

and $R_{\text{rms}} \triangleq \sqrt{\text{E}[\alpha^2]}$ is the rms envelope level.

i) Find the normalized threshold level $\rho_o$ at which the LCR reaches its maximum value.

ii) Explain why the LCR at $\rho$ decreases as $\rho$ deviates from $\rho_o$.

1b) 5 marks: Consider a cellular system with a carrier frequency of 2 GHz. Suppose that the user is in a vehicle travelling at 60 km/h. Assuming that the channel is characterized by 2D isotropic scattering, find

i) the LCR at the normalized level $\rho = -3$ dB.

ii) the AFD at the normalized level $\rho = -3$ dB.
2) The power delay profile for a WSSUS channel is given by

\[ \phi_{gg}(\tau) = \begin{cases} 
0.5[1 + \cos(\frac{2\pi\tau}{T})], & 0 \leq \tau \leq T/2 \\
0, & \text{otherwise}
\end{cases} \]

a) 3 marks: Find the channel frequency correlation function.

b) 4 marks: Calculate the mean delay and rms delay spread.

c) 3 marks: If \( T = 0.1 \) ms, determine whether the channel exhibits frequency-selective fading to the GSM system.
3) Cellular CDMA systems use soft handoff, where the transmissions to/from multiple base stations are combined to give a macro-diversity.

Here we consider the effects of path loss and shadowing and ignore multipath-fading. Suppose that the received signal power corresponding to the link with the $i$th base-station, $\Omega_{pi}$, has the probability density function

$$p_{\Omega_{pi}(\text{dBm})}(x) = \frac{1}{\sqrt{2\pi\sigma_{\Omega}}} \exp\left\{ -\frac{(x - \mu_{\Omega_{pi}(\text{dBm})})^2}{2\sigma_{\Omega}^2} \right\}.$$ 

where

$$\mu_{\Omega_{pi}(\text{dBm})} = E[\Omega_{pi}(\text{dBm})]$$

The $\Omega_{pi}$ are assumed to be statistically independent.

a) 5 marks: The reverse link uses selection combining such that the best base-station is always selected. In this case,

$$\Omega_{r}^{s}(\text{dBm}) = \max\{\Omega_{p1}(\text{dBm}), \ldots, \Omega_{pL}(\text{dBm})\}$$

An outage occurs if $\Omega_{r}^{s}(\text{dBm}) \leq \Omega_{th}(\text{dBm})$. What is the probability of outage?

b) 5 marks: The forward link uses coherent combining such that

$$\Omega_{p}^{mr}(\text{dBm}) = \Omega_{p1}(\text{dBm}) + \ldots + \Omega_{pL}(\text{dBm})$$

Again, an outage occurs if $\Omega_{p}^{mr}(\text{dBm}) \leq \Omega_{th}(\text{dBm})$. What is the probability of outage if

$$\mu_{\Omega_{p1}(\text{dBm})} = \mu_{\Omega_{p2}(\text{dBm})} = \cdots = \mu_{\Omega_{pL}(\text{dBm})}?$$
4) Consider the reception of a signal in the presence of a single co-channel interferer and neglect the effect of AWGN. The received signal power, \( C \), and interference power, \( I \), due to Rayleigh fading have the exponential distributions

\[
p_C(x) = \frac{1}{C} e^{-x/\bar{C}} \\
p_I(y) = \frac{1}{I} e^{-y/\bar{I}}
\]

where \( \bar{C} \) and \( \bar{I} \) are the average received signal power and interference power, respectively.

a) **5 marks:** Assuming that \( C \) and \( I \) are independent random variables, find the probability density function for the carrier-to-interference ratio

\[
\lambda = \frac{C}{I}.
\]

*Hint: If \( X \) and \( Y \) are independent random variables, then the probability density function of \( U = X/Y \) is

\[
p_U(u) = \int p_{XY}(v, v/u)|v/u^2|dv.
\]

b) **5 marks:** Now suppose that the system uses 2-branch selection diversity. The branches are independent and balanced (i.e., the distribution \( p_U(u) \) is the same for each branch. What is the probability density function of \( \lambda \) at the output of the selective combiner?
5) Suppose that a system uses selection diversity. The branches experience independent Rayleigh fading. However, the average received bit energy-to-noise ratio on each diversity branch is different, such that

\[ \bar{\gamma}_i = 2^{-i} \bar{\gamma}_o \quad i = 1, \ldots, L \]

a) 5 marks: Find the probability density function of the bit energy-to-noise ratio at the output of the selective combiner, denoted by \( \gamma_s \).

b) 5 marks: If DPSK modulation is used, write down an expression for the probability of bit error. Obtain a closed-form expression if possible; otherwise leave your expression in integral form.