Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Final Exam

Fall 2013

Monday December 9, 2:50pm - 5:40pm

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.
1) **a)** Consider the transmission of a bandpass signal having complex envelope \( \tilde{s}(t) \) on a channel such that the received complex envelope is

\[
\tilde{r}(t) = \alpha \tilde{s}(t) + \beta \tilde{s}(t - \tau_1),
\]

where \( \alpha \) and \( \beta \) are real valued.

i) (1 mark) Find the channel impulse response \( g(t, \tau) \).

ii) (2 marks) Find the channel magnitude response \( |G(t, f)| \).

iii) (2 marks) Find the channel phase response \( \angle G(t, f) \).

**b)** Consider the transmission of a bandpass signal having complex envelope \( \tilde{s}(t) \) on a channel such that the received complex envelope is

\[
\tilde{r}(t) = \alpha \tilde{s}(t) + \beta \tilde{s}(t) e^{j2\pi f_0 t},
\]

where \( \alpha \) and \( \beta \) are real valued.

i) (1 mark) Find the channel impulse response \( g(t, \tau) \).

ii) (2 marks) Find the channel magnitude response \( |G(t, f)| \).

iii) (2 marks) Find the channel phase response \( \angle G(t, f) \).
2) Consider the system shown in the figure below. A mobile station (MS) lies at a distance of 5 km, 10 km and 15 km from three base stations, BS\(_i\), \(i = 1, 2, 3\). BS\(_2\) is the serving base station, while BS\(_1\) and BS\(_3\) are co-channel base stations (co-channel interferers).

The propagation path loss follows the model

\[
\mu_{\Omega_p} (d) = \mu_{\Omega_p} (d_o) - 10 \beta \log_{10} (d/d_o) \quad \text{(dBm)}
\]

where \(\beta = 3.5\), and \(\mu_{\Omega_p}(d_o) = 1\) microwatt at \(d_o = 1\) km. Each radio link is affected independent log-normal shadowing with shadow standard deviation \(\sigma_{\Omega} = 8\) dB. Ignore envelope fading.

a) 5 marks: Obtain the probability density function of the total interfering power observed at the MS in decibel units.

b) 3 marks: What is the probability density function of the carrier-to-interference ratio observed at the MS in decibel units?

c) 2 marks: If the carrier-to-interference ratio must be greater than 6 dB for adequate radio link performance, what is the probability of outage?
3) A guard interval consisting of a cyclic prefix or cyclic suffix is used in OFDM systems to mitigate the effects of channel time dispersion.

a) 5 marks: Assess the cost of the cyclic prefix in terms of
i) bandwidth and/or data rate.
ii) transmitter power.

a) 5 marks: Suppose the guard interval of 500 ns is used. The data rate with 64-QAM modulation is 54 Mb/s. The power penalty due to the guard interval is to be kept less than 1 dB. What is the required value of $G$ (constrained to an integer) and the minimum possible OFDM block size $N$ (constrained to $2^k$ for some $k$)?
4) Consider binary, orthogonal signaling using non-coherent FSK modulation and demodulation. The probability of bit error for non-coherent FSK on an AWGN channel is

\[ P_b(\gamma_b) = \frac{1}{2} e^{-\gamma_b/2} \]

where \( \gamma_b = \alpha^2 E_b/N_o \) is the received bit energy-to-noise ratio. Derive the corresponding probability of bit error for

a) 5 marks: a flat Rayleigh fading channel.

b) 5 marks: a flat Ricean fading channel. An integral expression is acceptable, but it does reduce to closed form.
5) Consider a BPSK modulated system with simple repetition code and time interleaving, such that each data bit is transmitted $L$ times and each transmission experiences independent identically distributed (i.i.d.) Rayleigh fading. If symbol $\tilde{s}$ is transmitted, the corresponding $L$ correlator or matched filter outputs at the receiver are

$$\tilde{r}_k = \alpha_k \tilde{s} + \tilde{n}_k, \quad k = 1, \ldots, L$$

where $\tilde{s}$ is the transmitted BPSK symbol chosen from the alphabet $\{\pm \sqrt{2E}\}$, the $\alpha_k$ are i.i.d. Rayleigh random variables, and the $\tilde{n}_k$ are i.i.d. zero-mean complex Gaussian random variables with variance $\frac{1}{2}E[|\tilde{n}_k|^2] = N_0$.

a) 3 marks: One decoding strategy is to combine the $\tilde{r}_k$, $k = 1, \ldots, L$ using maximal ratio combining and then make a bit decision. What is the probability of decision error in terms of the average received bit energy-to-noise ratio, $\bar{\gamma}_b$?

b) 5 marks: Another decoding strategy is to make a hard decision as to which symbol was transmitted for each of the $\tilde{r}_k$, $k = 1, \ldots, L$, and then make a majority logic decision (assuming $L$ is odd) as to which data bit was transmitted, i.e., if more of the $L$ symbols comprising each bit are decided to be $+\sqrt{2E}$ than $-\sqrt{2E}$, then choose the data bit corresponding to symbol $+\sqrt{2E}$. What is the probability of decision error in terms of the average received bit energy-to-noise ratio, $\bar{\gamma}_b$?

c) 2 marks: Evaluate the probability of bit error in parts a) and b) when $L = 3$ and $\bar{\gamma}_b = 20$ dB.