Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Quiz

Thursday October 2, 2014

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.
1) Consider the transmission of a bandpass signal having complex envelope \( \tilde{s}(t) \) on a channel such that the received complex envelope is

\[
\tilde{r}(t) = \alpha \tilde{s}(t) + \beta \tilde{s}(t)e^{j2\pi ft},
\]

where \( \alpha \) and \( \beta \) are real valued.

i) (4 marks) Find the time-variant channel impulse response \( g(t, \tau) \) and time-variant channel transfer function \( G(t, f) \).

ii) (2 marks) Find the time-variant magnitude response \( |G(t, f)| \) and sketch.

iii) (2 marks) Find the time-variant phase response \( \angle G(t, f) \) and sketch.

\[ i) \quad \text{Since } \tilde{r}(t) = \tilde{s}(t) \ast g(t, \tau), \text{ we have} \]

\[
g(t, \tau) = (\alpha + \beta e^{j2\pi ft}) \tilde{s}(\tau)
\]

\[
G(t, f) = \mathcal{F}_t \{ g(t, \tau) \}
\]

\[
= \alpha + \beta e^{j2\pi ft} \quad 0 \leq f \leq \infty
\]

\[ ii) \quad |G(t, f)|^2 = G(t, f)G^*(t, f)
\]

\[
= \alpha^2 + \beta^2 + 2\alpha\beta \cos(2\pi ft)
\]

\[
|G(t, f)| = \sqrt{\alpha^2 + \beta^2 + 2\alpha\beta \cos(2\pi ft)}
\]

To sketch, set \( \alpha = 1, \beta = 1 \) (any \( \alpha, \beta \) will do)

\[
|G(t, f)| = \sqrt{2 + 2\cos(2\pi ft)}
\]

\[
= 2|\cos(n\pi ft)|
\]
iii) \[ G(t, f) = \alpha + \beta \cos 2\pi f dt + j \beta \sin 2\pi f dt \]

\[ \angle G(t, f) = \tan^{-1} \left\{ \frac{\beta \sin (2\pi f dt)}{\alpha + \beta \cos (2\pi f dt)} \right\} \]

To sketch, set \( \alpha = \beta = 1 \) (any choice will do)

\[ \angle G(t, f) = \tan^{-1} \left\{ \frac{\sin (2\pi f dt)}{1 + \cos (2\pi f dt)} \right\} \]
2) Consider the situation in the figure below, where the mobile station employs a bi-directional receiver antenna having a beam width of $\phi$, i.e.,

\[
G(\theta) = \begin{cases} 
1, & -\phi/2 \leq \theta \leq \phi/2 \text{ and } \pi - \phi/2 \leq \theta \leq \pi + \phi/2 \\
0, & \text{elsewhere}
\end{cases}
\]

Assume 2-D isotropic scattering with $p(\theta) = 1/(2\pi)$, $-\pi \leq \theta < \pi$.

![Diagram of beam of antenna](image)

a) In receiving a radio transmission at 850 MHz, a Doppler frequency of 20 to 60 Hz is observed. What is the beam width of the MS antenna, and how fast is the MS traveling?

b) Suppose that the MS antenna has a beam width of 13°. What is the level-crossing rate with respect to the rms envelope level, assuming that the MS is traveling at a speed of 30 km/h?

\[
a) \quad f_d = f_m \cos \theta \quad f_m = \frac{v}{\lambda}
\]

At 850 MHz \quad $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{850 \times 10^6} = 0.35m$

\[
v = f_m \lambda = 60 \times 0.35 = 21.17 \text{m/s} = 76 \text{Km/h}
\]

\[
f_d = f_m \cos \theta
\]

\[
20 = 60 \cos \theta \Rightarrow \theta = 70.5^\circ \Rightarrow \phi = 2\theta = 141^\circ
\]

b) Since $S_{SS}(f)$ is symmetric about $f=0$ and with $S=0$ (Rayleigh)

\[
\mathcal{P}(x, \xi) = \frac{1}{2\pi b_2} \sqrt{1 - \frac{x^2}{2b_2}} e^{-\frac{x^2}{2b_2}} e^{-\frac{\xi}{\phi}} = \mathcal{P}(x)\mathcal{P}(\xi)
\]
From (2.99) in text, with \( F_0 = 0 \),

\[
\begin{align*}
b_n &= (2\pi f_m)^n b_0 \int_0^{2\pi} \hat{\rho}(\theta) \cos^n \theta d\theta \quad ; \quad b_0 = \frac{B_0}{2} \\
b_2 &= (2\pi f_m)^2 b_0 \cdot 4 \int_0^{\theta_0} \frac{1}{4} \cos^2 \theta d\theta \quad ; \quad \theta_0 = \phi = 6.5^\circ \\
&= (2\pi f_m)^2 b_0 \cdot \frac{1}{\theta_0} \left[ \frac{1}{2} \frac{1}{4} \sin 2\theta \right]_0 \\
&= (2\pi f_m)^2 b_0 \cdot \frac{1}{\theta_0} \left[ \frac{1}{2} \frac{1}{4} \sin(2\theta_0) \right] \\
&= (2\pi f_m)^2 b_0 \left[ \frac{1}{2} + \frac{1}{4}\theta_0 \right]
\end{align*}
\]

\[
L_R = \int_0^R Q(R, \alpha) d\alpha
\]

\[
= \frac{R e^{-R^2/2b_0}}{b_0} \cdot \sqrt{\frac{1}{2\pi b_0}} \cdot b_2 = \frac{R e^{-R^2/2b_0}}{b_0} \cdot \sqrt{\frac{b_2}{2\pi}}
\]

\[
= \frac{R e^{-R^2/2b_0}}{b_0} \cdot \sqrt{2\pi f_m b_0} \left[ \frac{1}{2} + \frac{1}{4}\theta_0 \right]
\]

\[
= \frac{R e^{-R^2/2b_0}}{\sqrt{2b_0}} \cdot \sqrt{2\pi f_m} \cdot \sqrt{1 + \frac{1}{2\theta_0}} \sin(2\theta_0)
\]

\[
= \sqrt{2\pi f_m} \rho e^{-\rho^2} \sqrt{1 + \frac{1}{2\theta_0}} \sin(2\theta_0)
\]

At rms level \( \rho = 1 \)

At 850 MHz, \( f_m = v/\lambda = 8.33/0.35 = 23.8 \) Hz

\[
L_{R_{\text{rms}}} = \sqrt{2\pi} (23.8) e^{-1} \sqrt{1 + \frac{1}{0.22689}} \sin(22.689)
\]

\[= 30.97 \text{ crossings/second} \]
3) Consider the power delay profile shown below, which shows the relative power of the various taps or rays (in dB units) for some particular channel.

![Power Delay Profile](image)

Calculate the following:

a) 4 marks: mean delay of the channel, \( \mu_r \).

b) 4 marks: rms delay spread of the channel, \( \sigma_r \).

c) 2 marks: If the modulated symbol duration is 40 \( \mu s \), is the channel frequency selective? Why

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A) Assume the strongest ray has power 1 W. This is arbitrary, and any number will work. This gives the power delay profile below.

<table>
<thead>
<tr>
<th>Tap</th>
<th>Delay ((r_k))</th>
<th>Power ((P_k))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.01</td>
</tr>
</tbody>
</table>
\[ \mu_z = \frac{\sum_{k=1}^{d} \tau_k P_k}{\sum_k P_k} \]

\[ = \frac{0 \times 0.1 + 2 \times 1.0 + 4 \times 0.1 + 5 \times 0.01}{0.1 + 1.0 + 0.1 + 0.01} \]

\[ = 2.0248 \, \mu s \]

b) \[ \sigma_z = \sqrt{\frac{\sum_{k=1}^{d} (\tau_k - \mu_z)^2 P_k}{\sum_k P_k}} \]

\[ = \sqrt{\frac{(0 - \mu_z)^2 (0.1) + (2 - \mu_z)^2 (1) + (4 - \mu_z)^2 (0.1) + (5 - \mu_z)^2 (0.01)}{0.1 + 1.0 + 0.1 + 0.01}} \]

\[ = 0.85727 \, \mu s \]

c) Since \( \mu_z \ll T_s \) and \( T_E \ll T_s \) the channel is frequency non-selective, i.e. flat faded over the signal bandwidth.