ECE6604
PERSONAL & MOBILE COMMUNICATIONS

Lecture 2

Path Loss, Co-channel Interference, Link Budget
PROPAGATION OVER A FLAT SPECULAR SURFACE
• The length of the direct path is
\[ d_1 = \sqrt{d^2 + (h_b - h_m)^2} \]
and the length of the reflected path is
\[ d_2 = \sqrt{d^2 + (h_b + h_m)^2} \]

\( d \) = distance between mobile and base stations
\( h_b \) = base station antenna height
\( h_m \) = mobile station antenna height

• Given that \( d \gg h_b h_m \), we have \( d_1 \approx d \) and \( d_2 \approx d \).

• The carrier phase difference between the direct and reflected paths is
\[ \phi_1 - \phi_2 = \frac{2\pi}{\lambda_c} (d_1 - d_2) \]
Taking into account the phase difference, the received signal power is

\[ \mu_{\Omega_p} = \Omega_t \left( \frac{\lambda_c}{4\pi d} \right)^2 G_T G_R |1 + ae^{-jb}e^{j(\phi_2 - \phi_1)}|^2, \]

where \(a\) and \(b\) are the amplitude attenuation and phase change introduced by the flat reflecting surface.

If we assume a perfect specular reflection, then \(a = 1\) and \(b = \pi\) for small \(\theta\). Then

\[ \mu_{\Omega_p} = \Omega_t \left( \frac{\lambda_c}{4\pi d} \right)^2 G_T G_R |1 - e^{j(\frac{2\pi}{\lambda_c} \Delta d)}|^2 \]

\[ = \Omega_t \left( \frac{\lambda_c}{4\pi d} \right)^2 G_T G_R \sin^2 \left( \frac{2\pi}{\lambda_c} \Delta d \right) \]

where \(\Delta d = (d_1 - d_2)\).

Given that \(d \gg h_b\) and \(d \gg h_m\), and applying the approximation \(\sqrt{1 + x} \approx 1 + x/2\) for small \(x\), we have

\[ \Delta d \approx \frac{2h_b h_m}{d}. \]
Finally, the received envelope power is

\[ \mu_{\Omega_p} \approx 4 \Omega_t \left( \frac{\lambda_c}{4\pi d} \right)^2 G_T G_R \sin^2 \left( \frac{2\pi h_b h_m}{\lambda_c d} \right) \]

Under the condition that \( d \gg h_b h_m \), the above reduces to

\[ \mu_{\Omega_p} \approx \Omega_t G_T G_R \left( \frac{h_b h_m}{d^2} \right)^2 \]

where we have invoked the small angle approximation \( \sin x \approx x \) for small \( x \).

Propagation over a plane reflecting surface differs from free space propagation in two respects

- it is not frequency dependent
- signal strength decays with the with the fourth power of the distance, rather than the square of the distance.
Propagation path loss $L_p$ (dB) with distance over a flat reflecting surface; $h_b = 7.5 \text{ m}$, $h_m = 1.5 \text{ m}$, $f_c = 1800 \text{ MHz}$.

$$L_p = \left[ \left( \frac{\lambda_c}{4\pi d} \right)^2 \frac{4 \sin^2 \left( \frac{2\pi h_b h_m}{\lambda_c d} \right)}{\lambda_c d} \right]^{-1}$$
• In reality, the earth’s surface is curved and rough, and the signal strength typically decays with the inverse $\beta$ power of the distance, and the received power is

$$\Omega_p = k \frac{\Omega_t}{d^\beta}$$

where $k$ is a constant of proportionality. Expressed in units of dBm, the received power is

$$\Omega_p \text{ (dBm)} = 10\log_{10}(k) + \Omega_t \text{ (dBm)} - 10\beta\log_{10}(d)$$

• $\beta$ is called the path loss exponent. Typical values of $\beta$ are have been determined by empirical measurements for a variety of areas

<table>
<thead>
<tr>
<th>Terrain</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Space</td>
<td>2</td>
</tr>
<tr>
<td>Open Area</td>
<td>4.35</td>
</tr>
<tr>
<td>North American Suburban</td>
<td>3.84</td>
</tr>
<tr>
<td>North American Urban (Philadelphia)</td>
<td>3.68</td>
</tr>
<tr>
<td>North American Urban (Newark)</td>
<td>4.31</td>
</tr>
<tr>
<td>Japanese Urban (Tokyo)</td>
<td>3.05</td>
</tr>
</tbody>
</table>
Co-channel Interference

Worst case co-channel interference on the forward channel.
Worst Case Co-Channel Interference

• There are six co-channel base-stations, two at distance $D - R$, two at distance $D$, and two at distance of $D + R$.

• The worst case carrier-to-interference ratio is

$$\Lambda = \frac{1}{2} \left( \frac{R^{-\beta}}{(D - R)^{-\beta} + (D - R)^{-\beta} + (D + R)^{-\beta}} \right)$$

$$= \frac{1}{2} \left( \frac{1}{(\frac{D}{R} - 1)^{-\beta} + (\frac{D}{R})^{-\beta} + (\frac{D}{R} + 1)^{-\beta}} \right)$$

$$= \frac{1}{2} \left( \frac{1}{(\sqrt{3N} - 1)^{-\beta} + (\sqrt{3N})^{-\beta} + (\sqrt{3N} + 1)^{-\beta}} \right)$$

• Hence, for $\beta = 3.5$

$$\Lambda_{(dB)} = \begin{cases} 
14.3 \text{ dB} & \text{for } N = 7 \\
9.2 \text{ dB} & \text{for } N = 4 \\
6.3 \text{ dB} & \text{for } N = 3 
\end{cases}$$

– Shadows will introduce variations in the worst case $C/I$. 
Worst case co-channel interference on the forward channel with 120° cell sectoring.
• 120° cell sectoring reduces the number of co-channel base stations from six to two. The co-channel base stations are at distances $D$ and $D + 0.7R$.

• The carrier-to-interference ratio becomes

\[
\Lambda = \frac{R^{-\beta}}{D^{-\beta} + (D + 0.7R)^{-\beta}} \leq \frac{1}{(\frac{D}{R})^{-\beta} + (\frac{D}{R} + 0.7)^{-\beta}} = \frac{1}{(\sqrt{3N})^{-\beta} + (\sqrt{3N} + 0.7)^{-\beta}}
\]

• Hence

\[
\Lambda_{(dB)} = \begin{cases} 
21.1 \text{ dB} & \text{for } N = 7 \\
17.1 \text{ dB} & \text{for } N = 4 \\
15.0 \text{ dB} & \text{for } N = 3 
\end{cases}
\]

• For $N = 7$, 120° cell sectoring yields a 6.8 dB $C/I$ improvement over omni-cells.

• The minimum allowable cluster size is determined by the minimum $C/I$ requirement of the radio receiver. For example, if the radio receiver can operate at $\Lambda = 15.0 \text{ dB}$, then a 3/9 reuse cluster can be used (3/9 means 3 cells or 9 sectors per cluster).
Receiver Sensitivity

- Receiver sensitivity refers to the ability of the receiver to detect radio signals. We would like our radio receivers to be as sensitive as possible.

- Radio receivers must detect radio waves in the presence of noise.
  - External noise sources include atmospheric noise (e.g., lightning strikes), galactic noise, man made noise (e.g., automobile ignition noise).
  - Internal noise sources include thermal noise.

- The ratio of the desired signal power to thermal noise power before detection is commonly called the carrier-to-noise ratio, $\Gamma$.

- The parameter $\Gamma$ is a function of the communication link parameters including transmitted power (or effective isotropic radiated power (EIRP)), path loss, receiver antenna gain, and the effective input-noise temperature of the receiving system.

- The formula that relates the link parameters to $\Gamma$ is called the link budget.
Link Budget

- The link budget can be expressed in terms of the following parameters:

\[
\begin{align*}
\Omega_t &= \text{transmitted carrier power} \\
G_T &= \text{transmitter antenna gain} \\
L_p &= \text{path loss} \\
G_R &= \text{receiver antenna gain} \\
\Omega_p &= \text{received signal power} \\
E_c &= \text{received energy per modulated symbol} \\
T_o &= \text{receiving system noise temperature in degrees Kelvin} \\
B_w &= \text{receiver noise equivalent bandwidth} \\
N_o &= \text{white noise power spectral density} \\
R_c &= \text{modulated symbol rate} \\
k &= 1.38 \times 10^{-23} = \text{Boltzmann’s constant} \\
F &= \text{noise figure, typically about 3 dB} \\
L_{Rx} &= \text{receiver implementation losses} \\
L_I &= \text{losses due to system load (interference)} \\
M_{\text{shad}} &= \text{shadow margin} \\
G_{HO} &= \text{handoff gain} \\
S_{RX} &= \text{receiver sensitivity}
\end{align*}
\]
• The effective received carrier power is

\[ \Omega_p = \frac{\Omega_t G_T G_R}{L_{Rx} L_p} . \]

• The total input noise power to the receiver is

\[ N = kT_o B_w F \]

• Very often the following \( kT_o \) value at room temperature of 17 °C (290 °K) is used \( kT_o = -174 \text{ dBm/Hz} \),

• The received carrier-to-noise ratio defines the link budget

\[ \Gamma = \frac{\Omega_p}{N} = \frac{\Omega_t G_T G_R}{kT_o B_w F L_{Rx} L_p} . \]

• The carrier-to-noise ratio, \( \Gamma \), and modulated symbol energy-to-noise ratio, \( E_c/N_o \), are related as follows

\[ \frac{E_c}{N_o} = \Gamma \times \frac{B_w}{R_c} . \]

• Hence, we can rewrite the link budget as

\[ \frac{E_c}{N_o} = \frac{\Omega_t G_T G_R}{kT_o R_c F L_{Rx} L_p} . \]
• Converting into decibel units gives

\[ \frac{E_c}{N_o} (\text{dB}) = \Omega_t \text{ (dBm)} + G_T \text{ (dB)} + G_R \text{ (dB)} - kT_o(\text{dBm/Hz}) - R_c \text{ (dB Hz)} - F(\text{dB}) - L_{Rx} \text{ (dB)} - L_p \text{ (dB)} . \] (1)

• The receiver sensitivity is defined as

\[ S_{Rx} = L_{Rx} kT_o F(\frac{E_c}{N_o}) R_c \]

or converting to decibel units

\[ S_{Rx} \text{ (dBm)} = L_{Rx} \text{ (dB)} + kT_o(\text{dBm/Hz}) + F(\text{dB}) + \frac{E_c}{N_o} \text{ (dB)} + R_c \text{ (dB Hz)} . \]

• All parameters are usually fixed except for \( \frac{E_c}{N_o} \). The receiver sensitivity (in dBm) is determined by the minimum acceptable \( \frac{E_c}{N_o} \).

• Substituting the determined receiver sensitivity \( S_{Rx} \text{ (dBm)} \) into (1) and solving for \( L_p \text{ (dB)} \) gives the maximum allowable path loss

\[ L_{\text{max}} \text{ (dB)} = \Omega_t \text{ (dBm)} + G_T \text{ (dB)} + G_R \text{ (dB)} - S_{Rx} \text{ (dBm)} . \]