Utility-Based Joint Physical-MAC Layer Optimization in OFDM*

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Abstract—In this paper, joint physical-MAC layer optimization in OFDM systems using adaptive modulation and subcarrier allocation is investigated based on utility theory. Since utility qualifies the level of users’ satisfaction derived from the radio resources they occupy, it is ideal for a network optimization metric. We formulate cross-layer optimization problem as one that maximizes the sum of the utilities over all active users through rate adaptation and dynamic subcarrier allocation with the limited radio resource and time-varying wireless channel constraints. Two effective subcarrier allocation algorithms with low complexity are proposed to solve the constrained optimization problem. Simulation results demonstrate the significant improvement of utility-based cross-layer optimization in OFDM.

I. INTRODUCTION

The explosive growth of the Internet and the continuous increase in demand for all types of wireless services have been fueling the need for higher data rates, more aggressive spectrum reuse, and more efficient multiple access schemes.

Orthogonal frequency division multiplexing (OFDM) divides the entire channel into many orthogonal narrowband subchannels (subcarriers), and therefore, it supports high peak data rate by reducing the effect of intersymbol interference (ISI). Furthermore, in an OFDM system, different subcarriers can be allocated to different users to provide a flexible multiuser access scheme [1].

There is plenty of room to exploit the high degree of flexibility of radio resource management in the context of OFDM. First, since the channel frequency response of each user is frequency-selective, different subcarriers suffer from different fading levels. Using adaptive modulation [2] and [3], the transmitter can send higher data rates over the subcarriers with better condition. Therefore, adaptive modulation is able to adjust transmission symbol rate, so as to improve the throughput and simultaneously ensure an acceptable bit-error-rate (BER) in all subcarriers.

Second, despite using adaptive modulation, the deep fading on some subcarriers still leads to low channel capacity. However, since the channel characteristics for different users are almost mutually independent, the subcarriers experiencing deep fading for one user may not appear in deep fading for other users. In other words, each subcarrier is likely to be in good condition for some users in a multiuser OFDM network. Consequently, adaptive subcarrier allocation should be able to exploit knowledge of the channel state information to make the best subcarrier assignment, thereby improving the system performance [4] and [5].

Subcarrier allocation is actually a scheduling problem in a sharing resource environment. From a whole network point of view, network performance should be evaluated in terms of the degree to which network satisfies the service requirements of users’ applications, rather than in terms of system-centric quantities like throughput, outage probability, packet drop rate, power, etc. [6]. Utility offers a tangible metric for network provisioning when application performance is the key concern. More specifically, utility theory provides a means to formulate the gain of a quality measurement when a user is assigned certain resources. In this paper, we use utility to measure the level of users’ satisfaction and total system utility as the primary optimization objective.

With the help of utility functions, cross-layer optimization, including subcarrier rate adaption and subcarrier assignment, is proposed in this paper. It is based on maximizing the sum of the utilities over all active users in time-varying wireless environments. Furthermore, efficient frequency assignment algorithms are presented to solve the utility-based optimization. Specifically, for best effort traffic, we simulate achievable total utilities by using different schemes. Our results show that cross-layer optimization can significantly improve the network performance. The gain comes from dynamic subcarrier allocation, which increases with the number of users in the cell.

This paper is organized as follows. In Section II, we present the channel model, general properties of utility functions, and the model of adaptive modulation. In Section III, we formulate the cross-layer optimization problem and propose subcarrier assignment algorithms to obtain the optimal solution. Then, in Section IV, we develop efficient subcarrier assignment algorithms for the discrete frequency case. Finally, we show simulation results in Section V, and then summarize the paper in Section VI.

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II. SYSTEM MODEL

In this section, we present the channel model, concepts related to utility function, and the model of adaptive modulation, which will help formulate the utility-based cross-layer optimization problem.

A. Channel Frequency Response

The complex baseband representation of a mobile wireless channel impulse response can be described by

\[ h(t, \tau) = \sum_k \gamma_k(t) \delta(\tau - \tau_k), \]

where \( \tau_k \) is the delay of the \( k \)-th path and \( \gamma_k(t) \) is its corresponding complex amplitude. In addition, \( \gamma_k(t) \)'s are assumed to be wide-sense stationary narrowband complex Gaussian processes, and independent for different paths. The frequency response of the above channel impulse response is given by

\[ H(t, f) = \int_{-\infty}^{+\infty} h(t, \tau) e^{-j2\pi f \tau} \, d\tau = \sum_k \gamma_k(t) e^{-j2\pi f \tau_k}. \tag{1} \]

It is assumed that the channel fading rate is slow enough that the channel amplitudes are constant in each OFDM block, and that the perfect channel state information is known at the base-station. Therefore, the parameter \( t \) in \( H(t, f) \) can be ignored.

The downlink transmission in a single-cell OFDM system is actually a set of multiple narrowband broadcast channels. The basestation transmits information simultaneously to all users, but the channels for all users are independent. The channel frequency response corresponding to user \( i \) is denoted by \( H_i(f) \).

B. Utility Functions

Utility function maps the network resources a user utilizes into the level of satisfaction. In almost all wireless applications, reliable data transmission rate is the most important factor to determine the satisfaction of users. Therefore, the utility function \( U(r) \) should be a nondecreasing function of \( r \).

In particular, when \( U(r) = r \), the utility is just throughput. Furthermore, most traditional system optimization objectives can also be regarded as some special cases of utility functions. Thus, this work can be regarded as a general network optimization theory.

Utility functions, determined by applications, serve as the optimization objective of adaptive physical and media access control (MAC) layer techniques. As a consequence, using different utility functions can optimize limited radio resources for different applications. Utility functions can build a bridge to connect the physical, MAC, and higher layers.

C. Adaptive Modulation

Adaptive modulation provides the system with the ability to match the effective throughput to the channel conditions of different subcarriers of a specific user. The signal-to-noise-ratio (SNR) of user \( i \), SNR \( i(f) \), at frequency \( f \) is given by

\[ \text{SNR}_i(f) = \frac{P_i(f) | H_i(f)|^2}{N_i(f)}, \]

where \( N_i(f) \) is the one-sided noise power spectral density for user \( i \), \( P_i(f) \) is the one-sided signal power density transmitted at frequency \( f \) for user \( i \), which is assumed to be 1 (no power allocation) in this paper. Let \( c_i(f) \) be the achievable throughput of user \( i \) at frequency \( f \) under a given BER level, which is assumed to be a continuous number at first. It is a function of SNR \( i(f) \) expressed as [7]

\[ c_i(f) = \log_2(1 + \beta \text{SNR}_i(f)) \text{ [bps/Hz]}, \tag{2} \]

where \( \beta \) is a constant related to BER by

\[ \beta = \frac{1.5}{-\ln(5\text{BER})}. \]

In the discrete rate case, the transmission rate is only varied within a fixed set. When the variable MQAM is assumed to be used, the discrete rate with a given BER is obtained by [7]

\[ c_i(f) = \text{floor}[\log_2(1 + \beta \text{SNR}_i(f))]. \tag{3} \]

III. ADAPTIVE SUBCARRIER ALLOCATION WITH ADAPTIVE MODULATION

In this section, we investigate the case of continuous frequency scheduling problem where there are infinite orthogonal subcarriers in all frequency resources, or the bandwidth of each orthogonal subcarrier \( \Delta f \to 0 \). It can be regarded as an extreme situation of OFDM. It can help us to obtain a deep insight into this problem and provide a useful guide to design scheduling algorithms since the continuous programming problem can be regarded as a relaxation of integer programming problem. Therefore, it has both theoretical and practical importance.

Consider a single cell consisting of \( M \) users, and let \( \mathcal{M} \) denote the set of users, that is \( \mathcal{M} = \{1, 2, \ldots, M\} \). The whole frequency band, \([0, B]\), will be divided into several non-overlapping frequency sets that are assigned to all users, where \( B \) is the total bandwidth of the system. Define \( D_i \) as the frequency set occupied by user \( i \). Then the transmission throughput of user \( i \) can be calculated by

\[ r_i = \int_{D_i} c_i(f) \, df. \tag{4} \]

As mentioned before, the goal of subcarrier assignment is to maximize the total utility. Hence, the constrained optimization
problem can be expressed as follows:

$$\begin{align*}
\text{maximize} & \quad \sum_{i \in \mathcal{M}} U_i(r_i) \\
\text{subject to} & \quad \bigcup_{i \in \mathcal{M}} D_i = [0, B] \\
& \quad D_i \cap D_j = \emptyset, \quad i \neq j \quad \forall i, j \in \mathcal{M}. \quad (7)
\end{align*}$$

The optimal subcarrier allocation, $D_i^*$'s, have an important property, which is stated in the following theorem.

**Theorem 1:** If the set of $r_i^*$, $i \in \mathcal{M}$ is optimal, then the optimal frequency set for user $i$, $D_i^*$, satisfies

$$D_i^* = \bigcap_{j \in \mathcal{M} \setminus \{i\}} \{f \in [0 : B] : c_j(f) \leq \alpha_{ij}^*\}. \quad (8)$$

where

$$\alpha_{ij}^* \triangleq \frac{U_i^*(r_j^*)}{U_j^*(r_i^*)} \quad (9)$$

$$r_j^* = \int_{D_j^*} c_i(f) \, df \quad (10)$$

**Proof:** If $D_i^*$'s are optimal to maximize $\sum_{i \in \mathcal{M}} U_i(r_i)$, any change of allocation will not increase the total utility. Let $(f_i - \frac{1}{2} \Delta f, f_i + \frac{1}{2} \Delta f) \in D_i^*$. If $(f_i - \frac{1}{2} \Delta f, f_i + \frac{1}{2} \Delta f)$ is assigned to another user (e.g., user $j$), then the data rate of user $i$ will be decreased by $\Delta r_i = c_i(f_i) \Delta f$, while the data rate of user $j$ will be increased by $\Delta r_j = c_j(f_i) \Delta f$. But the new sum of utilities will be equal to or less than the optimal one. Thus, we have

$$\sum_{m \in \mathcal{M} \setminus \{i,j\}} U_m(r_m) + U_i(r_i^* - \Delta r_i) + U_j(r_j^* + \Delta r_j) - \sum_{m \in \mathcal{M}} U_m(r_m) \leq 0,$$

which is equivalent to

$$U_i(r_i^* - \Delta r_i) + U_j(r_j^* + \Delta r_j) - U_i(r_i^*) - U_j(r_j^*) \leq 0.$$

Divided by $\Delta r_j$, it becomes

$$\frac{U_j(r_j^* + \Delta r_j) - U_j(r_j^*)}{\Delta r_j} \leq \frac{U_i(r_i^*) - U_i(r_i^* - \Delta r_i)}{\Delta r_i}.$$

Using the relationship $\Delta r_j = \frac{c_i(f_i)}{c_j(f_i)} \Delta r_i$, we have

$$\frac{U_j(r_j^* + \Delta r_j) - U_j(r_j^*)}{\Delta r_j} \leq \frac{c_i(f_i)}{c_j(f_i)} \frac{U_i(r_i^*) - U_i(r_i^* - \Delta r_i)}{\Delta r_i}.$$

Let $\Delta f \to 0$, then $\Delta r_j \to 0$, and $r_i \to 0$, and

$$\lim_{\Delta r_j \to 0} \frac{U_j(r_j^* + \Delta r_j) - U_j(r_j^*)}{\Delta r_j} \leq \frac{c_i(f_i)}{c_j(f_i)} \lim_{\Delta r_i \to 0} \frac{U_i(r_i^*) - U_i(r_i^* - \Delta r_i)}{\Delta r_i}. \quad (11)$$

It can be expressed as follows if $U_i(r_i), i \in \mathcal{M}$ are differentiable

$$\frac{c_j(f_i)}{c_i(f_i)} \leq \frac{U'_i(r_i^*)}{U'_j(r_j^*)} \quad f_i \in D_i^* \quad (12)$$

With all $j$'s, $j \in \mathcal{M} \setminus \{i\}$, therefore, the optimal frequency allocation for user $i$ $D_i^*$ can be expressed as (8).

**Theorem 1** implies that the problem of searching the optimal subcarrier allocation is actually equivalent to finding $\alpha_{ij}^*$s, a set of optimal decision thresholds. Using constraints (6) and (7), we can get an efficient algorithm to find out $D_i$'s, when $\alpha$ is given, where

$$\alpha = [\alpha_{12}, \alpha_{23}, \ldots, \alpha_{1M}, \alpha_{23}, \alpha_{24}, \ldots, \alpha_{2M}, \ldots, \alpha_{M-1M}]^T.$$

It is shown as follows.

**Algorithm 1:** (Finding $D_i$'s when $\alpha$ is given)

$$\begin{align*}
D_{\text{assigned}} &= \emptyset; \\
& \text{for } i = 1 : M - 1 \\
& \text{for } j = i + 1 : M \\
& \quad D_i = \{f \in (D_i - D_i \cap D_{\text{assigned}}) : \frac{c_i(f)}{c_j(f)} \leq \alpha_{ij}\}; \\
& \quad D_{\text{assigned}} = D_{\text{assigned}} \cup D_i; \\
& \quad D_M = [0, B] - D_{\text{assigned}}.
\end{align*}$$

The optimal $\alpha$, can be found by the following iterative algorithm:

**Algorithm 2:** (Iterative algorithm for $D_i^*$'s)

$$\begin{align*}
\text{Initialize } \alpha; \\
& \text{Given } \mu \in (0, 1); \quad \epsilon > 0; \\
& \text{While } \| \alpha_{n+1} - \alpha_n \| > \epsilon \\
& \quad \text{Get } D_i, i \in \mathcal{M} \text{ from } \alpha_n \text{ using Algorithm 1}; \\
& \quad \text{Calculate } r_i, i \in \mathcal{M} \text{ using (4)}; \\
& \quad \alpha_{n+1} = \alpha_n + \mu[\alpha(r_1, r_2, \ldots, r_M) - \alpha_n]; \\
& \text{end}
\end{align*}$$

With algorithms 1 and 2, we can effectively get the optimal frequency assignment.

**IV. DISCRETE IMPLEMENTATION**

In a practical OFDM system, there are only finite subcarriers. Inspired by Theorem 1, we can develop an efficient algorithm of integer programming for subcarrier allocation.

Assume $K$ to be the number of subcarriers, and $K$ denote to be the set of subcarriers’ indices, that is, $K = \{1, 2, \ldots, K\}$. If the bandwidth of each subcarrier $\Delta f$ is small enough that SNR $i(f)$ is nearly constant over each subcarrier, the achievable transmission data rate of user $i$ over subcarrier $k$ is given by

$$c_i(k) = \log_2(1 + \beta \text{SNR}_i(k)).$$
Instead of integral (4), the transmission throughput of user $i$ becomes

$$r_i = \sum_{k \in D_i} c_i(k).$$

For the optimization problem, optimization objective (5) and constraint (7) remain unchanged, but constraint (6) should be changed to

$$\bigcup_{i \in M} D_i = K.$$

Note that the frequency set assigned to user $i$, $D_i$, is a union of some closed intervals in $[0, B]$ in the continuous frequency case, but is a subset of $K$ in the discrete frequency case.

Although Algorithms 1 and 2 can be used to solve this problem, but the convergence of Algorithm 2 is slow and greatly depends on the initial values and $\mu$. Therefore, we will develop more efficient algorithms suitable for this integer programming problem.

### A. Efficient Algorithm for Two-User Case

Here we consider two-user case to gain some insight of the optimization problem. In this case, each subcarrier should be assigned to either user 1 or user 2. For this combinatorial optimization problem, there are $2^K$ choices to assign $K$ subcarriers. It looks like a nondeterministic polynomial-hard (NP-hard) problem. However, since the continuous optimization problem (5) to (7) is a relaxation of this integer programming, the property of optimal solution described by Theorem 1 is still valid. Therefore, the optimal subcarrier allocation and optimal data rates have the following relationships:

$$D_1^* = \{k \in K : \frac{c_2(k)}{c_1(k)} \leq \alpha_{12}(T)\},$$

$$D_2^* = K - D_1^*.$$  \hspace{1cm} (13)

Obviously, we have Lemma 1.

**Lemma 1:** In the two-user case, if subcarrier $i$ is assigned to user 1, and $\frac{c_2(j)}{c_1(j)} \leq \frac{c_2(i)}{c_1(i)}$, then subcarrier $j$ must be assigned to user 1.

From Lemma 1, after $\{\frac{c_2(k)}{c_1(k)} \mid k \in K\}$ are sorted in increasing order, there are only $K + 1$ possible choices, including the two extreme cases in which all subcarriers are assigned to user 1 or user 2. Therefore, after investigating the structure, we find out that this problem is actually not a NP-hard one. In the two-user case, some sorting and searching algorithms can effectively solve this combinatorial optimization problem. Specially, the goal of searching is to find out the optimal assignment threshold $\alpha_{12}$ of increasingly sorted series $\frac{c_2(k)}{c_1(k)}$. For a given threshold $T$, we can get a new assignment, in which subcarriers satisfying $\frac{c_2(k)}{c_1(k)} > T$ are assigned to user 2, and the rest to user 1. With utility function, we can calculate the new threshold $\alpha_{12}(T)$. If $T - \alpha_{12}(T) > 0$, it indicates that $T$ is too high for the optimal threshold; otherwise, $T$ is lower than the optimal one. Thus $T - \alpha_{12}(T)$ can be used as the directive sign for binary search.

**Algorithm 3:** (Optimal subcarrier allocation for the two-user case)

```
Sort $\frac{c_2(k)}{c_1(k)}, \ k \in K$ in increasing order
Get thresholds: $T(k), \ k \in \{1 : K + 1\}$ in increasing order
low = 1; high = $K + 1$
while high - low > 0
   center = $[(low + high)/2]$;
   if $T(center) - \alpha_{12}(T(center)) > 0$
      high = center;
   else if low = center + 1; end
end
Choose the best one between $T(low)$ and $T(high)$
```

The above algorithm is very effective. The computation complexity of sorting is about $K \log_2(K)$, and that of binary search is only $\log_2(K)$ [8].

### B. Algorithm for M-User Case

For the $M$-user case, we can update the subcarrier assignment of every two users iteratively using the subcarrier allocation algorithm for the two-user case. Obviously, the computation complexity of this algorithm is nearly $(M - 1)^2 K \log_2(K)$, which appears efficient compared to the number of choices of this combinatorial optimization problem, $K^M$.

### V. Simulation Results

Different types of applications have different utility functions. For best effort application, the survey results in [9] indicate that the log function fits the subjective survey very well,

$$U(r) = 0.16 + 0.8 \ln(r - 0.3),$$  \hspace{1cm} (15)

where $r$ is in unit of kbits/sec. With the utility function (15), the subcarrier assignment scheme is optimized to maximize the performance of best effort applications.

In this simulation, channel is assumed to have the bad urban (BU) delay profiles [10] and suffer from shadowing with 8.5 dB standard deviation. Let the acceptable BER be $10^{-3}$. For OFDM, there are 128 subcarriers. To be able to compare those results properly, we let the average bandwidth per user, $B/M$, be 60 kHz.

Figure 1 shows some numerical results of different resource allocation schemes in the continuous rate adaptation case. Numerical results are plotted in terms of average utility per user. Notes that the fixed subcarrier allocation has almost the same performance for different numbers of users. It can be seen from Figure 1 that adaptive subcarrier allocation offers a significant system gain, which increases as the number of users increases. For example, in order for the average utility to reach 3, the system gains of using adaptive subcarrier allocation are 2 dB, 3
dB, 5 dB for the case of 2 users, 4 users, 16 users, respectively, compared to the scheme that only exploits adaptive modulation. This is because with the increase of the number of active users, the possibility of each subcarrier in deep fading for all users becomes lower.

Figure 2 shows the performance of discrete rate adaptation. The adaptive subcarrier allocation can save 6 dB power to achieve 3 in utility in the 16-user case, compared to the fixed subcarrier allocation. The simulation results illustrate that adaptive subcarrier allocation has more advantage over fixed subcarrier allocation scheme in the discrete rate adaptation case than in the continuous.

VI. CONCLUSIONS

Cross-layer optimization design is needed to deal with the major challenges to high-data-rate wireless networks: limited radio resources, unreliable wireless channel, and the increasing demand for quality of service. In the physical and MAC layers, adaptive modulation and dynamic subcarrier allocation are proposed for OFDM systems. In higher layers, utility function can also characterize different types of applications since it establishes the relationship between quality of service and assigned resources. Therefore, utility offers a tangible performance metric for intelligent resource management in both physical and MAC layers. Utility-based cross-layer optimization can balance the applications’ demands, available resources, and wireless channel conditions to maximize the total network utility.

The problem of adaptive modulation and subcarrier allocation is formulated on the basis of maximizing the total system utility. Two efficient algorithms are proposed to solve this joint constrained optimization problem for continuous and discrete frequency cases, respectively. As regards the finite subcarrier number $K$, the algorithm complexity is about $(M - 1)^2 \log_2 K$, where $M$ is the number of users. Simulation results show that cross-layer optimization can significantly enhance the system performance. The gain coming from dynamic subcarrier allocation increases with the number of users in the cell.

Utility-based cross-layer optimization is a new frontier in wireless networks. There are many issues remaining to be explored. We are currently investigating utility-based joint subcarrier assignment and power allocation, as well as interference environments in multiple cells.

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