Example 2.10 Construct the SFG for the block diagram shown in Fig.2-19 .

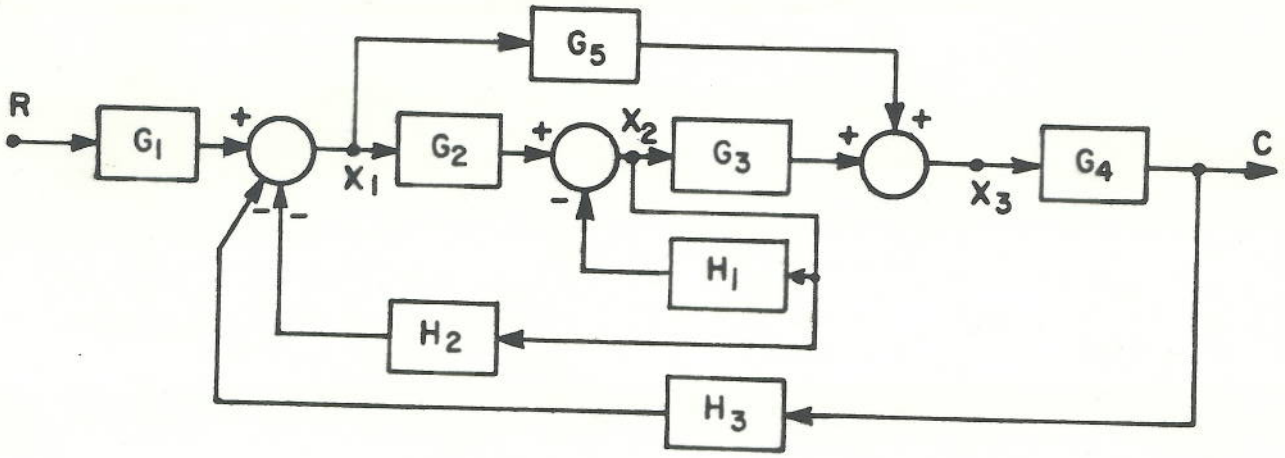


Fig.2-19 Block diagram for Example 2-10

The signal flow graph corresponding to the block diagram of Fig.2-19 is shown in Fig.2-20.

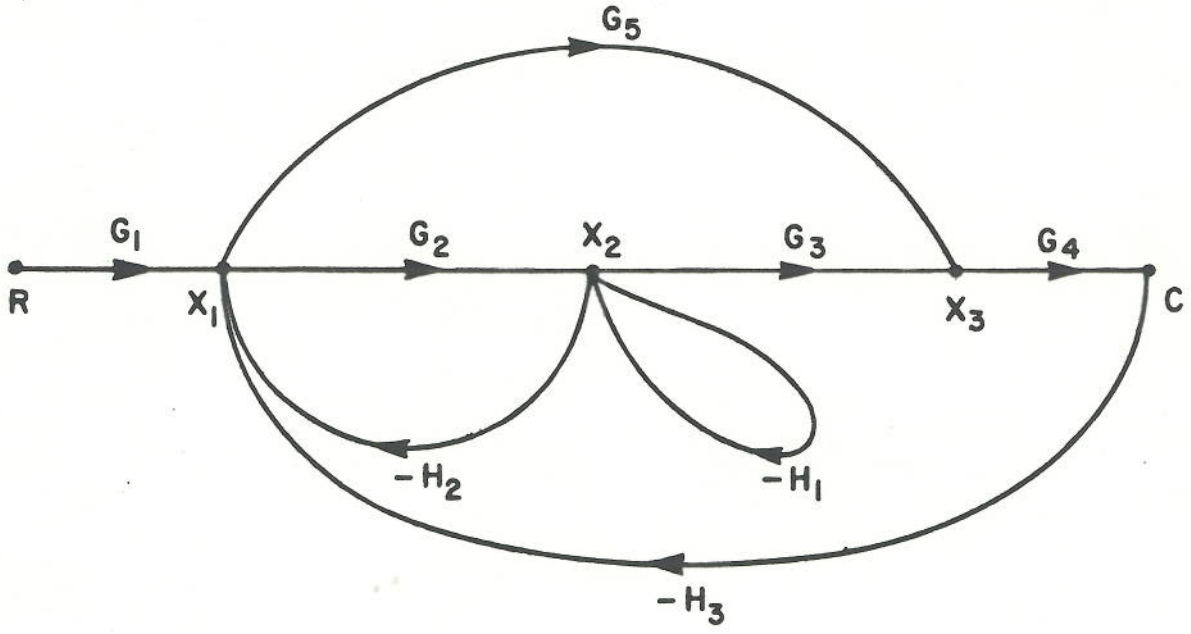


Fig.2-20 SFG equivalent to block diagram of Fig.2-18 .

* * *

Mason's Formula

Before presenting the Mason's formula to determine the transfer function between the output and input nodes of a SFG, we define some new concepts. Consider the SFG of Fig.(2-20).

A forward path is a trajectory directed from the input node to the output node without passing twice through a node. In the SFG of Fig. 2-20 there are two forward paths with gains: $G_1G_4G_5$ and $G_1G_2G_3G_4$.

A loop is a closed path following the arrows. There are four loops in Fig. 2-20 with gains: $-H_1$ (self-loop), $-G_2H_2$, $-G_2G_3G_4H_3$ and $-G_4G_5H_3$.

A nontouching graph to a forward path is the portion of the SFG that is not touching the forward path. For instance, the nontouching graph for forward path $G_1G_4G_5$ is the self-loop with gain $-H_1$. Does the forward path $G_1G_2G_3G_4$ have a nontouching graph?

Using Mason's formula, the transfer function C(s)/R(s) is given by

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{N} P_i \Delta_i}{\Delta} \tag{2-84}$$

where

N : number of forward paths

P_i : gain of the i^{th} forward path

$$\Delta = 1 - \sum_j L'_j + \sum_k L''_k - \sum_q L'''_q \pm \cdots \tag{2-85}$$

L'_j : gain of the j^{th} loop

L''_k : gain product of two nontouching loops (no nodes in common)

L'''_q : gain product of three nontouching loops

Δ_i : Δ corresponding to that portion of the graph nontouching to path of gain P_i.

Example 2.11 For the SFG shown in Fig. 2-20, find the transfer function C(s)/R(s) using Mason's formula.

Step 1 There are two forward paths, as shown in Fig. 2-21(a),

$N = 2$: $P_1 = G_1G_2G_3G_4$, $P_2 = G_1G_4G_5$

Step 2 There are four single loops (Fig. 2-21(b)),

$L'_1 = -H_1$, $L'_2 = -G_2H_2$, $L'_3 = -G_2G_3G_4H_3$, $L'_4 = -G_3G_5H_3$

Step 3 As shown in Fig. 2-21(c), we have only two nontouching loops with product gain

$L''_1 = (-H_1)(-G_4G_5H_3) = G_4G_5H_1H_3$

step 4 There is no group of three or more nontouching loops.

step 5 Δ is given by

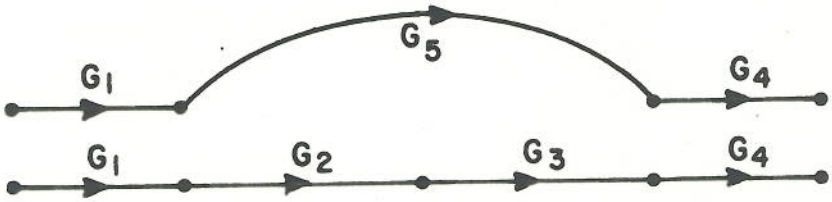
$$\Delta = 1 + H_1 + G_2H_2 + G_2G_3G_4H_3 + G_4G_5H_3 + G_4G_5H_1H_3$$

step 6 There is no portion of graph nontouching to forward path number 1. Therefore, $\Delta_1 = 1$. The nontouching graph for forward path number 2 is the self-loop of gain $-H_1$. Thus, $\Delta_2 = 1 + H_1$.

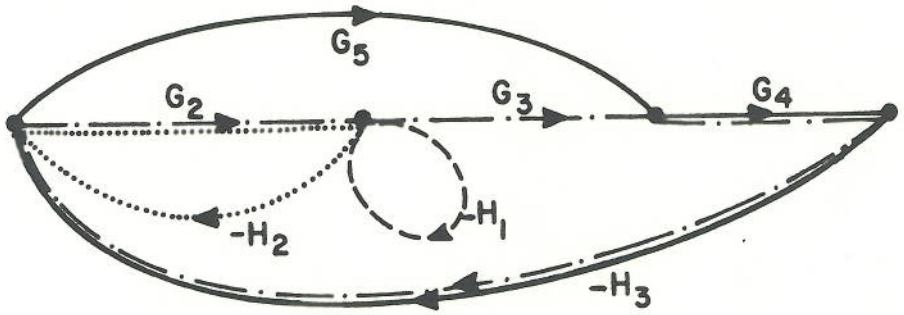
step 7 The closed-loop function is

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1(G_2G_3 + G_5 + H_1G_5)\,G_4}{1 + H_1 + G_2(H_2 + G_3G_4H_3) + G_4G_5H_3(1 + H_1)} \quad . \qquad (2\text{-}86)$$

(a)



(b)



(c)

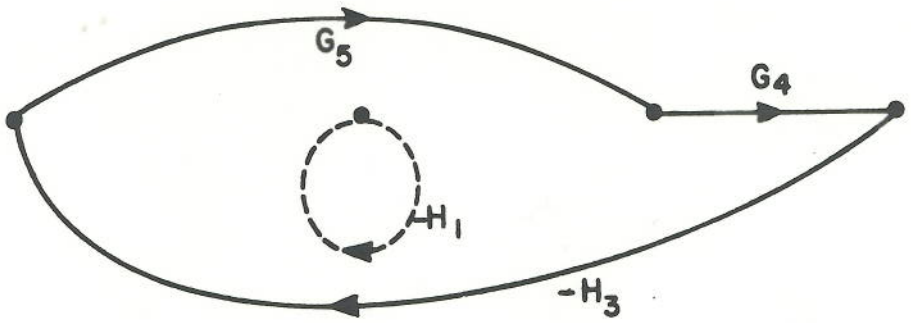


Fig.2-21 Forward paths and loops of SGF of Fig.2-20 .

* * *