8.4 ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM)

All of the modulation techniques discussed so far are *single-carrier* modulation techniques that employ a single RF carrier. *Multi-carrier* modulation techniques on the other hand transmit data symbols in parallel on multiple subcarriers. A block of N data symbols, each of duration T_s, is converted into a block of N parallel data symbols, each of duration $T = NT_s$. The N parallel data symbols modulate N subcarriers that are spaced uniformly in frequency $\Delta_f = 1/T$ Hz apart. To describe and work with OFDM waveforms, it is most convenient to use the complex envelope representation for bandpass signals. The OFDM complex envelope is given by

$$\tilde{s}(t) = \sum_n b(t - nT, \mathbf{x}_n) \ , \tag{8.61}$$

where

$$b(t, \mathbf{x}_n) = \frac{\sqrt{2E}}{T} u_T(t) \sum_{k=0}^{N-1} x_{n,k} \mathrm{e}^{j\frac{2\pi k t}{T}} \tag{8.62}$$

n is the block index, k is the subcarrier index, N is the number of sub-carriers, and $\mathbf{x}_n = \{x_{n,0},\ x_{n,1},\ \ldots,\ x_{n,N-1}\}$ is the nth data symbol block. Note that the amplitude shaping pulse used on each subcarrier is the rectangular pulse $g(t) = \frac{\sqrt{2E}}{T} u_T(t)$. The complex-valued data symbols $x_{n,k} = x^I_{n,k} + j x^Q_{n,k}$ are usually chosen from a QAM or PSK signal constellation. The OFDM bandpass signal has the quadrature representation

$$\begin{aligned} s(t) &= \frac{\sqrt{2E}}{T} \sum_n u_T(t - nT) \\ &\quad \times \sum_{k=0}^{N-1} x^I_{n,k} \cos\left(2\pi\left(f_c + \frac{k}{T}\right)t\right) - x^Q_{n,k} \sin\left(2\pi\left(f_c + \frac{k}{T}\right)t\right) \end{aligned} \tag{8.63}$$

and the envelope-phase representation

$$s(t) = \frac{\sqrt{2E}}{T} \sum_n u_T(t - nT) \sum_{k=0}^{N-1} |x_{n,k}| \cos\left(2\pi\left(f_c + \frac{k}{T}\right)t + \phi_{n,k}\right) \tag{8.64}$$

where $\phi_{n,k} = \mathrm{Tan}^{-1}(x^Q_{n,k}/x^I_{n,k})$. From the quadrature representation in (8.63) it is apparent that E represents the energy in the elemental bandpass waveform

$$s(t) = \frac{\sqrt{2E}}{T} u_T(t) \cos\left(2\pi\left(f_c + \frac{k}{T}\right)t\right)$$

or

$$s(t) = \frac{\sqrt{2E}}{T} u_T(t) \sin\left(2\pi\left(f_c + \frac{k}{T}\right)t\right)$$

From the envelope-phase representation in (8.64), consider the waveforms corresponding to two different subcarriers in the nth OFDM symbol

$$\begin{aligned} s_{n,k}(t) &= \frac{\sqrt{2E}}{T} u_T(t) |x_{n,k}| \cos\left(2\pi\left(f_c + \frac{k}{T}\right)t + \phi_{n,k}\right) \\ s_{n,\ell}(t) &= \frac{\sqrt{2E}}{T} u_T(t) |x_{n,\ell}| \cos\left(2\pi\left(f_c + \frac{\ell}{T}\right)t + \phi_{n,\ell}\right) \end{aligned}$$

Separating adjacent OFDM subcarriers in frequency by $\Delta_f = 1/T$ Hz ensures that the corresponding OFDM subchannels are mutually orthogonal regardless of the set random phases $\{\phi_{n,k}\}$ that are imparted by the data modulation (see Problem **??**).

A cyclic extension (or guard interval) is usually appended to the OFDM waveform in (8.61) and (8.62) to combat ISI as will be explained later. The cyclic extension can be in the form of a cyclic prefix, a cyclic suffix, or both. With a cyclic suffix, the OFDM complex envelope becomes

$$\tilde{s}_g(t) = \begin{cases} \tilde{s}(t) & , \ 0 \leq t \leq T \\ \tilde{s}(t-T) & , \ T \leq t \leq (1+\alpha_g)T \end{cases} \quad , \tag{8.65}$$

where $\alpha_g T$ is the length of the guard interval and $\tilde{s}(t)$ is defined in (8.61) and (8.62). Note that the segments of the waveform $\tilde{s}_g(t)$ in the intervals $[0, \alpha_g T)$ and $[T, T+\alpha_g T)$ are identical. The OFDM complex envelope with a cyclic suffix can be rewritten in the form

$$\tilde{s}_g(t) = \sum_n b(t - nT_g, \mathbf{x}_n) \ , \tag{8.66}$$

where

$$\begin{aligned} b(t, \mathbf{x}_n) &= u_T(t) \sum_{k=0}^{N-1} x_{n,k} \mathrm{e}^{j\frac{2\pi k t}{T}} + u_{\alpha_g T}(t-T) \sum_{k=0}^{N-1} x_{n,k} \mathrm{e}^{j\frac{2\pi k (t-T)}{T}} \\ &= u_T(t) \sum_{k=0}^{N-1} x_{n,k} \mathrm{e}^{j\frac{2\pi k t}{T}} + u_{\alpha_g T}(t-T) \sum_{k=0}^{N-1} x_{n,k} \mathrm{e}^{j\frac{2\pi k t}{T}} \\ &= u_{T_g}(t) \sum_{k=0}^{N-1} x_{n,k} \mathrm{e}^{j\frac{2\pi k t}{T}} \ , \end{aligned} \tag{8.67}$$

and $T_g = (1+\alpha_g)T$ is the OFDM symbol period with the addition of the guard interval. Likewise, with a cyclic prefix, the OFDM complex envelope is given by (8.66) with

$$\tilde{s}_g(t) = \begin{cases} \tilde{s}(t+T) & , \ -\alpha_g T \leq t \leq 0 \\ \tilde{s}(t) & , \ 0 \leq t \leq T \end{cases} \quad , \tag{8.68}$$

and

$$\begin{aligned} b(t, \mathbf{x}_n) &= u_{\alpha_g T}(t+\alpha_g T) \sum_{k=0}^{N-1} x_{n,k} \mathrm{e}^{j\frac{2\pi k (t+T)}{T}} + u_T(t) \sum_{k=0}^{N-1} x_{n,k} \mathrm{e}^{j\frac{2\pi k t}{T}} \\ &= u_{\alpha_g T}(t+\alpha_g T) \sum_{k=0}^{N-1} x_{n,k} \mathrm{e}^{j\frac{2\pi k t}{T}} + u_T(t) \sum_{k=0}^{N-1} x_{n,k} \mathrm{e}^{j\frac{2\pi k t}{T}} \\ &= u_{T_g}(t+\alpha_g T) \sum_{k=0}^{N-1} x_{n,k} \mathrm{e}^{j\frac{2\pi k t}{T}} \end{aligned} \tag{8.69}$$

Finally, the cyclic extension can be divided between a cyclic prefix and cyclic suffix. Let $\alpha_g = \alpha_p + \alpha_s$, where $\alpha_p T$ and $\alpha_s T$ are the length of the cyclic prefix and cyclic suffix, respectively. Then the OFDM complex envelope is given by (8.66) with

$$\tilde{s}_g(t) = \begin{cases} \tilde{s}(t+T) & , \ -\alpha_p T \leq t \leq 0 \\ \tilde{s}(t) & , \ 0 \leq t \leq T \\ \tilde{s}(t-T) & , \ t \leq t \leq \alpha_s T \end{cases} \quad , \tag{8.70}$$

and

$$\begin{aligned} b(t,\mathbf{x}_n) &= u_{\alpha_p T}(t+\alpha_p T)\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{j\frac{2\pi k(t+T)}{T}} + u_T(t)\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{j\frac{2\pi kt}{T}} \\ &\quad + u_{\alpha_s T}(t-T)\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{j\frac{2\pi k(t-T)}{T}} \\ &= u_{\alpha_p T}(t+\alpha_p T)\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{j\frac{2\pi kt}{T}} + u_T(t)\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{j\frac{2\pi kt}{T}} \\ &\quad + u_{\alpha_s T}(t-T)\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{j\frac{2\pi kt}{T}} \\ &= u_{T_g}(t+\alpha_p T)\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{j\frac{2\pi kt}{T}} \end{aligned} \tag{8.71}$$

8.4.1 DFT-Based OFDM Baseband Modulator

A key advantage of using OFDM is that the baseband modulator can be implemented by using an inverse discrete-time Fourier transform (IDFT). Later, we will see that the baseband demodulator can be implemented by using a discrete-time Fourier transform (DFT). In practice, the computationally efficient fast Fourier transform (FFT) and inverse fast Fourier transform (IFFT) is used to implement the DFT and IDFT, respectively. The FFT and IFFT are very common signal processing functions.

Consider the OFDM complex envelope without guard interval, as defined by (8.61) and (8.62). During the interval $nT \le t \le (n+1)T$, the OFDM complex envelope has the form

$$\begin{aligned} \tilde{s}(t) &= \sqrt{E}u_T(t-nT)\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{\frac{j2\pi k(t-nT)}{T}} \\ &= \sqrt{E}u_T(t-nT)\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{\frac{j2\pi kt}{NT_s}}\ ,\ \ nT \le t \le (n+1)T\ , \end{aligned} \tag{8.72}$$

where we have made the substitution $T = NT_s$. Now suppose that the complex envelope in (8.72) is sampled at T_s second intervals to yield the sample sequence

$$X_{n,m} = \tilde{s}(mT_s) = \sqrt{E}\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{\frac{j2\pi km}{N}}\ ,\quad m = 0,\ 1,\ \ldots,\ N-1\ . \tag{8.73}$$

Note that there are N samples in each OFDM symbol, since $T = NT_s$. Observe that the vector $\mathbf{X}_n = \{X_{n,m}\}_{m=0}^{N-1}$ is the IDFT of the vector $\sqrt{E}\mathbf{x}_n = \sqrt{E}\{x_{n,k}\}_{k=0}^{N-1}$. Contrary to conventional notation used when working with transforms, with OFDM it is customary to use the lower case vector $\sqrt{E}\mathbf{x}_n$ to represent frequency domain coefficients, while the upper case vector $\mathbf{X}_n$ is used to represent time domain coefficients.

As mentioned earlier, a cyclic extension (or guard interval) is usually added to the OFDM waveform as described in (8.66) and (8.67) to combat ISI. When a cyclic suffix is used, the corresponding sample sequence is

$$X^g_{n,m} = X_{n,(m)_N} \tag{8.74}$$

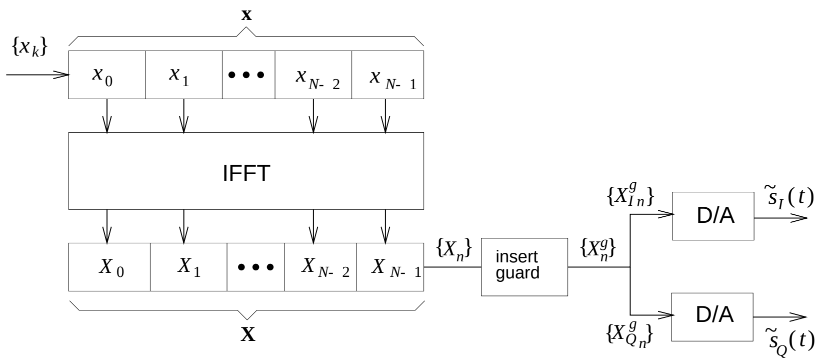

Figure 8.9 Block diagram of IDFT-based baseband OFDM modulator with guard interval insertion and digital-to-analog conversion.

$$= \sqrt{E}\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{\frac{j2\pi km}{N}} \ , \quad m = 0,\ 1,\ \ldots,\ N+G-1 \ , \tag{8.75}$$

where G is the length of the guard interval in samples, and $(m)_N$ is the remainder after dividing m by N. This gives the vector $\mathbf{X}_n^g = \{X_{n,m}^g\}_{m=0}^{N+G-1}$, where the values in the first and last G coordinates of the vector $\mathbf{X}_n^g$ are the same. Likewise, when a cyclic prefix is used, the corresponding sample sequence is

$$X_{n,m}^g = X_{n,(m)_N} \tag{8.76}$$

$$= \sqrt{E}\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{\frac{j2\pi km}{N}} \ , \quad m = -G,\ \ldots, -1,\ 0,\ 1,\ \ldots,\ N-1 \ . \tag{8.77}$$

This yields the vector $\mathbf{X}_n^g = \{X_{n,m}^g\}_{m=-G}^{N-1}$, where again the first and last G coordinates of the vector $\mathbf{X}_n^g$ are the same. The sampling interval after insertion of the guard interval, T_s^g, must be compressed in time such that

$$(N+G)T_s^g = NT_s$$

The OFDM complex envelope can be generated by splitting the complex-valued output vector $\mathbf{X}_m$ into its real and imaginary parts, $\Re(\mathbf{X}_n)$ and $\Im(\mathbf{X}_n)$, respectively. The sequences $\{\Re(X_{n_m})\}$ and $\{\Im(X_{n_m})\}$ are then input to a pair of balanced digital-to-analog converters (DACs) to generate the real and imaginary components $\tilde{s}_I(t)$ and $\tilde{s}_Q(t)$, respectively, of the complex envelope $\tilde{s}(t)$, during the time interval $nT \leq t \leq (n+1)T$. As shown in Fig. 8.9, the OFDM baseband modulator consists of an IFFT operation, followed by guard interval insertion and digital-to-analog conversion.

It is instructive at this stage to realize that the waveform generated by using the IDFT OFDM baseband modulator is *not* exactly the same as the waveform generated from the analog waveform definition of OFDM. Consider for example, the OFDM waveform without a cyclic guard in (8.61) and (8.62). The analog waveform definition uses the rectangular amplitude shaping pulse $u_T(t)$ that is strictly time-limited to T seconds. Later, we will see that the corresponding OFDM power spectrum has infinite bandwidth, and any finite sampling rate of the complex envelope will necessarily lead to aliasing and imperfect reconstruction. With the IDFT OFDM baseband modulator, we apply the IDFT/IFFT outputs to a pair of balanced DACs as explained earlier. However, the ideal DAC is an ideal low-pass filter with cutoff frequency $1/(2T_s)$, with a corresponding noncausal impulse response given by $h(t) = \mathrm{sinc}(t/T_s)$. Since the ideal DAC is nonrealizable, a causal reconstruction filter is used instead. However, such a filter will necessarily generate a waveform that is not strictly bandlimited, thus leading to side lobes. In conclusion, the side lobes of the analog OFDM waveform are inherent in the waveform itself due to rectangular pulse shaping, whereas the side lobes with the IDFT implementation arises from the non-ideal reconstruction filter, i.e., the DAC.

8.4.1.1 Error Probability for OFDM The OFDM baseband demodulator is usually implemented by using a fast Fourier transform (FFT), as discussed in Section 8.4. Following the development in Section 8.4, suppose that the discrete-time sequence $\mathbf{X}_n^g = \{X_{n,m}^g\}_{m=0}^{N+G-1}$ is passed through a balanced pair of digital-to-analog converters (DACs), as shown in Fig. 8.9. The real and imaginary components of the resulting complex envelope are applied to a quadrature modulator as shown in Fig. **??**, and the resulting bandpass OFDM waveform is transmitted over an AWGN channel.

The receiver first uses a quadrature demodulator as shown in Fig. 6.3 to extract the received complex envelope $\tilde{r}(t) = \tilde{r}_I(t) + j\tilde{r}_Q(t)$. Suppose that the quadrature components $\tilde{r}_I(t)$ and $\tilde{r}_Q(t)$ are each passed through an ideal anti-aliasing filter (ideal low-pass filter) having a cutoff frequency $1/(2T_s^g)$ followed by an analog-to-digital converter (ADC) as shown in Fig. 8.10. This produces the received complex-valued sample sequence $\mathbf{R}_n^g = \{R_{n,m}^g\}_{m=0}^{N+G-1}$, where

$$R_{n,m}^g = X_{n,m}^g + \tilde{n}_{n,m} \quad m = 0, 1, \ldots, N + G - 1 \ , \tag{8.78}$$

and the $\tilde{n}_{n,m}$ are complex-valued Gaussian noise samples. For an ideal anti-aliasing filter having a cutoff frequency $1/(2T_s^g)$, the $\tilde{n}_{n,m}$ are independent zero-mean complex Gaussian random variables with variance $\sigma^2 = \frac{1}{2}\mathrm{E}[|\tilde{n}_{n,m}|^2] = N_o/T_s^g$, where $T_s^g = NT_s/(N+G)$.

Assuming a cyclic suffix as discussed in Sect. 8.4.1, the receiver first removes the guard interval according to

$$R_{n,m} = R_{n,G+(m-G)_N}^g, \ 0 \le m \le N - 1 \ , \tag{8.79}$$

where $(m)_N$ is the residue of m modulo N. This operation simply deletes the first G values of $R_{n,m}^g$ in (8.88) and replaces them with the last G values of $R_{n,m}^g$, resulting in a new length-N sequence $R_{n,m}$ in (8.79). Demodulation is then performed by computing the FFT on the block $\mathbf{R}_n = \{R_{n,m}\}_{m=0}^{N-1}$ to yield the vector $\mathbf{z}_n = \{z_{n,k}\}_{k=0}^{N-1}$ of N decision variables

$$\begin{aligned} z_{n,k} &= \frac{1}{N}\sum_{m=0}^{N-1} R_{n,m}\mathrm{e}^{-\frac{j2\pi km}{N}} \\ &= \sqrt{E}x_{n,k} + \nu_{n,k} \ , \ \ k = 0, \ldots, N-1 \ , \end{aligned} \tag{8.80}$$

where the noise terms are given by

$$\nu_{n,k} = \frac{1}{N}\sum_{m=0}^{N-1} \tilde{n}_{n,m}\mathrm{e}^{-\frac{j2\pi km}{N}} \ , \ \ k = 0, \ldots, N-1 \ . \tag{8.81}$$

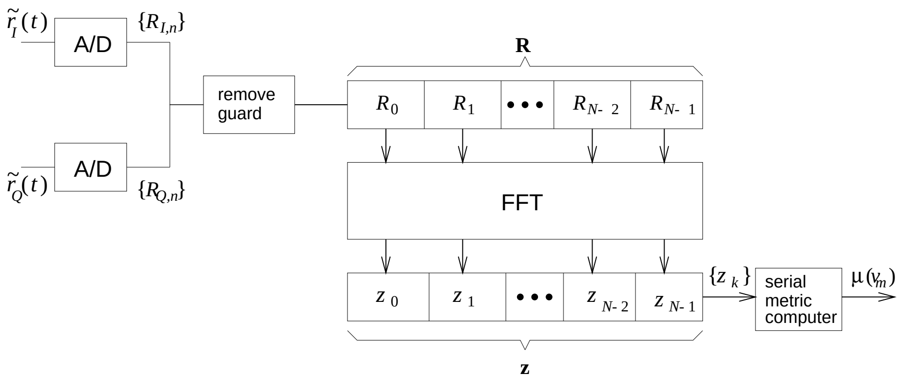

Figure 8.10 Block diagram of OFDM receiver.

It can be shown that the $\nu_{n,k}$ are zero mean complex Gaussian random variables with covariance

$$\phi_{j,k} = \frac{1}{2}\mathrm{E}[\nu_{n,j}\nu_{n,k}^*] = \frac{N_o}{NT_s^g}\delta_{jk} \quad . \tag{8.82}$$

Hence, the $z_{n,k}$ are independent Gaussian random variables with mean $\sqrt{E}x_{n,k}$ and variance N_o/NT_s^g. To be consistent with our earlier results for PSK and QAM signals, we can multiply the $z_{n,k}$ for convenience by the scalar $\sqrt{NT_s^g}$. Such scaling gives

$$\tilde{z}_{n,k} = \sqrt{EN/(N+G)}x_{n,k} + \tilde{\nu}_{n,k} \quad , \tag{8.83}$$

where the $\tilde{\nu}_{n,k}$ are i.i.d. zero-mean Gaussian random variables with variance N_o. Notice that $\sqrt{EN/(N+G)}x_{n,k}$ is equal to the complex signal vector that is transmitted on the ith sub-carrier, where the term $N/(N+G)$ represents the loss in effective symbol energy due to the insertion of the cyclic guard interval. For *each* of the $\tilde{z}_{n,k}$, the receiver decides in favor of the complex-valued data symbol $x_{n,k}$ that minimizes the squared Euclidean distance

$$\mu(\tilde{s}_{n,k}) = \|\tilde{z}_{n,k} - \sqrt{EN/(N+G)}x_{n,k}\|^2 \ , \quad k = 0, \ldots, N-1 \ . \tag{8.84}$$

Thus, for each OFDM block, N symbol decisions are made, one for each of the N sub-carriers. It is apparent from (8.83) that the probability of symbol error is identical to that achieved with independent modulation on each of the sub-carriers. This is expected, because the sub-carriers are mutually orthogonal.

8.4.1.2 OFDM on ISI Channels Consider an OFDM system where the number of sub-carriers N is chosen to be large enough so that the channel transfer function $T(t,f)$ is essentially constant across sub-bands of width $1/T$. Then

$$T(t,k) \doteq T(t,k\Delta_f), \quad k\Delta_f - 1/(2T) \le f \le k\Delta_f + (1/(2T) \ , \tag{8.85}$$

and no equalization is necessary because the ISI is negligible. Viewing the problem another way, if the block length N is chosen so that $N \gg L$, where L is the length of the discrete-time channel impulse response, then the ISI will only affect a small fraction of the symbol transmitted on each sub-carrier. Weinstein and Ebert [4] came up with an ingenious solution, whereby they inserted a guard interval in the form of a length-G cyclic prefix or cyclic suffix to each IDFT output vector $\mathbf{X}_n = \{X_{n,m}\}$. In fact, if the discrete-time channel impulse response has duration $L \leq G$, a cyclic guard interval can completely remove the ISI in a very efficient fashion as we now describe.

Suppose that the IDFT output vector $\mathbf{X}_n = \{X_{n,m}\}_{m=0}^{N-1}$ is appended with a cyclic suffix to yield the vector $\mathbf{X}_n^g = \{X_{n,m}^g\}_{m=0}^{N+G-1}$, where

$$\begin{aligned} X_{n,m}^g &= X_{n,(m)_N} && (8.86) \\ &= \sqrt{E}\sum_{k=0}^{N-1} x_{n,k}\mathrm{e}^{\frac{j2\pi km}{N}} \ , \quad m = 0,\ 1,\ \ldots,\ N+G-1 \ , && (8.87) \end{aligned}$$

G is the length of the guard interval in samples, and $(m)_N$ is the residue of m modulo N. To maintain the data rate $R_s = 1/T_s$, the DAC in the transmitter is clocked with rate $R_s^g = \frac{N+G}{N}R_s$, due to the insertion of the cyclic guard interval.

Consider a time-invariant ISI channel with impulse response $g(t)$. The combination of the ADC, waveform channel $g(t)$, anti aliasing filter, and DAC yields an overall discrete-time channel with sampled impulse response $\mathbf{g} = \{g_m\}_{m=0}^{L}$, where L is the length of the discrete-time channel impulse response. The discrete-time *linear convolution* of the transmitted sequence $\{\mathbf{X}_n^g\}$ with the discrete-time channel produces the discrete-time received sequence $\{R_{n,m}^g\}$, where

$$R_{n,m}^g = \begin{cases} \sum_{i=0}^{m} g_i X_{n,m-i}^g + \sum_{i=m+1}^{L} g_i X_{n-1,N+G+m-i}^g + \tilde{n}_{n,m} \ , & 0 \leq m < L \\ \sum_{i=0}^{L} g_i X_{n,m-i}^g + \tilde{n}_{n,m} \ , & L \leq m \leq N+G-1 \end{cases} . \tag{8.88}$$

Notice that the first L samples of $R_{n,m}^g$ in OFDM block n are corrupted by ISI from the previous OFDM block $n-1$.

To remove the ISI introduced by the channel, the first G received samples $\{R_{n,m}^g\}_{m=0}^{G-1}$ are discarded and replaced with the last G received samples $\{R_{n,m}^g\}_{m=N}^{N+G-1}$, as shown in Fig. 8.11. If the length of the guard interval satisfies $G \geq L$, then we obtain the received sequence

$$\begin{aligned} R_{n,m} &= R_{n,G+(m-G)_N}^g \\ &= \sum_{i=0}^{L} g_i X_{n,(m-i)_N} + \tilde{n}_{n,(m-i)_N} \ , \ \ 0 \leq m \leq N-1 \ . && (8.89) \end{aligned}$$

Note that the first term in (8.89) represents a *circular convolution* of the transmitted sequence $\mathbf{X}_n = \{X_{n,m}\}$ with the discrete-time channel $\mathbf{g} = \{g_m\}_{m=0}^{L}$.

As shown in Fig. 8.10, the OFDM baseband demodulator computes the DFT (in practice the FFT) of the length-N vector $\mathbf{R}_n$ after the guard interval has been removed. This yields the DFT/FFT output vector

$$\begin{aligned} z_{n,i} &= \frac{1}{N}\sum_{m=0}^{N-1} R_{n,m}\mathrm{e}^{-j\frac{2\pi mi}{N}} \\ &= T_i\sqrt{E}x_{n,i} + \nu_{n,i} \ , \ \ 0 \leq i \leq N-1 \ , && (8.90) \end{aligned}$$

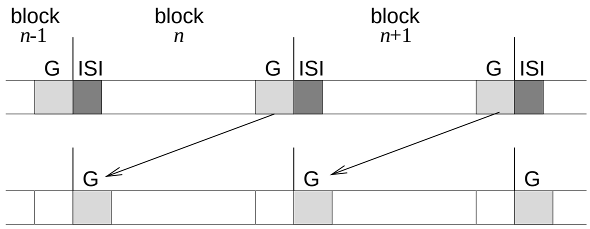

Figure 8.11 Removal of ISI by using the cyclic suffix.

where

$$T_i = \sum_{m=0}^{L} g_m \mathrm{e}^{-j\frac{2\pi m i}{N}} \tag{8.91}$$

is the FFT of the channel impulse response $\mathbf{g} = \{g_m\}_{m=0}^{L}$, and the noise samples $\{\nu_{n,i}\}$ are i.i.d with zero-mean and variance $N_o/(NT_s^g)$. Note that $\mathbf{T} = \{T_i\}_{i=0}^{N-1}$ is the DFT of the zero-padded sequence $\mathbf{g} = \{g_m\}_{m=0}^{N-1}$ and is equal to the sampled frequency response of the channel. To be consistent with our earlier results, we can multiply the $z_{n,i}$ for convenience by the scalar $\sqrt{NT_s^g}$. By following the same argument used in Sect. 8.4.1.1, such scaling gives

$$\tilde{z}_{n,i} = \sqrt{EN/(N+G)}T_i x_{n,i} + \tilde{\nu}_{n,i} \quad i = 0, \ldots, N-1 \ , \tag{8.92}$$

where the $\tilde{\nu}_{n,i}$ are i.i.d. zero-mean Gaussian random variables with variance N_o. Observe that each decision variable $\tilde{z}_{n,i}$ depends only on the corresponding data symbol $x_{n,i}$ and, therefore, the ISI has been completely removed. Once again, for *each* of the $\tilde{z}_{n,i}$, the receiver decides in favor of the complex-valued data symbol $x_{n,k}$ that minimizes the squared Euclidean distance

$$\mu(\tilde{s}_m) = \|\tilde{z}_{n,i} - \sqrt{EN/(N+G)}T_i x_{n,i}\|^2 \ . \tag{8.93}$$

Thus, for each OFDM block, N symbol decisions must be made, one for each of the N sub-carriers.