GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
Final Examination - Spring 2008
EE 4601: Communication Systems

Aids Allowed: Course textbook
one $8\frac{1}{2} \times 11$ two-side crib sheet
Attempt all questions
All questions are of equal value

DATE: Monday April 28, 2008.

TIME: 8:00am - 10:50am
INSTRUCTOR: Professor G.L. Stüber

1) Random Processes: Let $s(t)$ be a Gaussian random process with zero mean and autocorrelation function

$$\phi_{ss}(\tau) = 10^{-5} B_w \text{sinc}^2(B_w \tau), \qquad B_w = 10^6$$

The process is applied to filter having the impulse response

$$h(t) = B\text{sinc}(2Bt), \qquad B = 10^5 \text{ Hz}$$

to generate the output $y(t) = s(t) * h(t)$.

a) 4 marks: Sketch and fully dimension the power spectral density at the output of the filter.

b) 2 marks: What is the probability density function of the random variable $y(t_1)$ at any time instant t_1?

c) 4 marks: What is the probability that the output $y(t)$ lies in the range $[-0.1, 0.1]$?

2) ISI Channels: Suppose that the frequency response characteristic of a channel can be approximated by

$$H(f) = \begin{cases} 1 + \alpha \cos(2\pi f t_o), & |\alpha| < 1, |f| \leq W \\ 0, & \text{otherwise} \end{cases}$$

where W Hz is the channel bandwidth. An input signal $s(t)$ whose spectrum is bandlimited to W Hz is passed through the channel.

a) 5 marks: Find the signal $y(t) = s(t) * h(t)$ at the output of the channel.

b) 3 marks: Suppose the received signal $y(t)$ is passed through a filter matched to $s(t)$. Determine the output of the matched filter at $t = kT, k = 0, \pm 1, \pm 2, \ldots$, where T is the symbol duration.

c) 2 marks: What is the overall sampled pulse $p_k = p(kT)$ if $t_o = T$?

3) Digital Modulation: Suppose that binary modulation is used with the following waveforms

$$\begin{aligned} s_1(t) &= Au_T(t), \qquad 0 \le t \le T \\ s_2(t) &= B\sin(2\pi t/T), \qquad 0 \le t \le T \end{aligned}$$

where $u_T(t) = 1$ for $0 \le t \le T$ and 0 otherwise.

a) 2 marks: Find a set of orthonormal basis functions for this signal set.

b) 3 marks: Draw the signal space diagram.

c) 5 marks: Assuming that each message waveform is transmitted with equal probability on an AWGN channel with two-sided noise power spectral density of $N_o/2$ watts/Hz, what is the probability of bit error? Express your result in terms of the bit-energy-to-noise ratio and specify the bit energy-to-noise ratio in terms of A, B, T, and N_o.

4) Signal Space: Suppose that the following signal vectors are used on an AWGN channel with two-sided noise power spectral density of $N_o/2$ watts/Hz.

$$\begin{aligned} \mathbf{s}_1 &= (a, 2a, a) \\ \mathbf{s}_2 &= (a, 0, a) \\ \mathbf{s}_3 &= (-a, 2a, a) \\ \mathbf{s}_4 &= (-a, 0, a) \end{aligned}$$

a) 7 marks: Find the *exact* probability of *symbol* error in terms of the average *bit* energy-to-noise ratio. *Hint: use rotations and translations of the signal set as appropriate.*

b) 3 marks: Construct a set of 4 time domain waveforms that correspond to these signal vectors. *Hint: the solution is not unique.*

5) General Suppose that BSPK modulation is used on an AWGN channel with two-sided noise power spectral density $N_o/2$ watts/Hz. The two possible transmitted waveforms in each bit interval are

$$s_1(t) = -s_2(t) = \sqrt{\frac{2E}{T}}\cos(2\pi f_c t), 0 \leq t \leq T$$

where E is the bit energy and T is the bit duration. The receiver uses a coherent correlation detector. However, the frequency of receiver local oscillator is $f_c + \Delta f$ Hz.

a) 5 marks: Derive an expression showing how the output of the receiver's correlator is affected by the frequency difference Δf. Assume $f_c T \gg 1$.

b) 5 marks: Derive an expression for the probability of bit error in terms of Δf and E_b/N_o.