GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
Final Examination - Spring 2009
EE 4601: Communication Systems

Aids Allowed: Course textbook
one $8\frac{1}{2} \times 11$ two-side crib sheet
Attempt all questions
All questions are of equal value

DATE: Wednesday April 29, 2009.

TIME: 8:00am - 10:50am
INSTRUCTOR: Professor G.L. Stüber

1) Random Processes: Let X and Y be statistically independent Gaussian-distributed random variables, each with zero mean and unit variance. Define the Gaussian process

$$Z(t) = X\cos(2\pi t) + Y\sin(2\pi t)$$

a) 4 marks: Determine the autocorrelation function for this random process.

b) 2 marks: Determine the power spectral density for this random process.

c) 2 marks: Determine the joint probability density function of the random variables $Z(t_1)$ and $Z(t_2)$ obtained by observing $Z(t)$ at times t_1 and t_2, respectively.

d) 2 marks: Is the process $Z(t)$ stationary? Why?

2) ISI Channels: Suppose that a binary data sequence $\mathbf{a} : a_k \in \{-1 + 1\}$ is transmitted on a channel having the following overall discrete-time impulse response

$$\mathbf{p} = (p_0, p_1, p_2) = (1.0, 0.25, -0.25)$$

and $p_i = 0$, $i < 0, i > 2$. The sample at the input to the decision device at the kth epoch is

$$y_k = p_0 a_k + \sum_{n \neq k} a_n p_{k-n} + n_k$$

where the n_k are independent zero-mean Gaussian random variables with variance σ^2.

Assume that the a_k are independent and take on values -1 and $+1$ with equal probability, and assume that no equalization is used. The decision device makes the decision

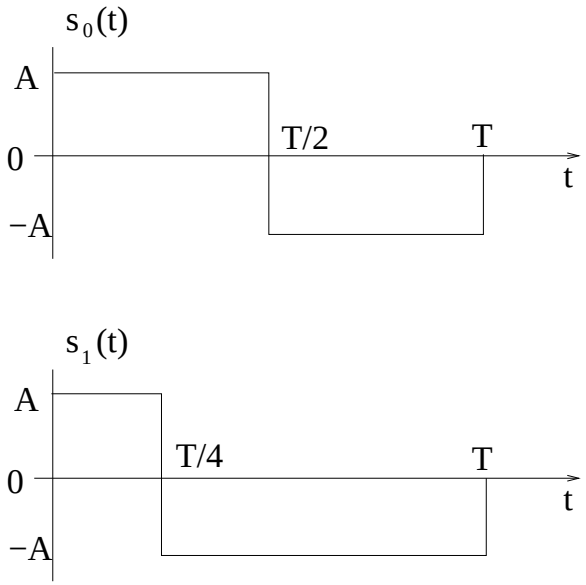
$$\hat{a}_k = \begin{cases} -1 & \text{if } y_k < 0 \\ +1 & \text{if } y_k > 0 \end{cases}$$

a) 4 marks What are the possible values for the ISI term, and what are the probabilities that these values occur?

b) 4 marks Write down an expression for the probability of bit error, using the Gaussian Q function. *Hint:* use the theorem of total probability.

c) 2 marks Now suppose that a zero forcing equalizer is used. Would the zero-forcing solution minimize the peak distortion after equalization?

3) Signal Space: Consider the pair of waveforms $s_0(t)$ and $s_1(t)$ shown in the figure below



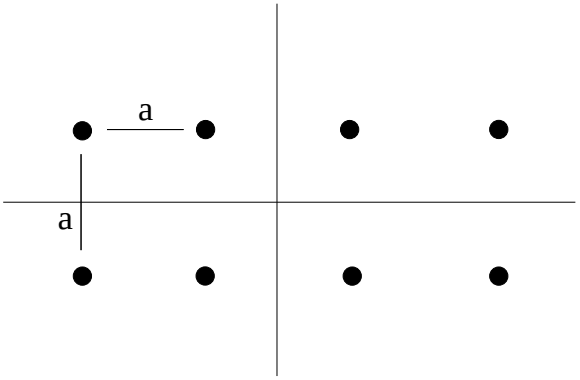
a) 4 marks: Using the Gram-Schmidt orthonormalization procedure, find a complete set of orthonormal basis functions.

b) 2 marks: Find the signal vectors $\mathbf{s}_0$ and $\mathbf{s}_1$ and plot them.

c) 3 marks: Find the Euclidean distance $d_{12} = \|\mathbf{s}_0 - \mathbf{s}_1\|$.

d) 3 marks: If the pair of waveforms is used for binary modulation on an AWGN channel, write down an expression for the probability of bit error in terms of the bit energy-to-noise ratio E_b/N_o.

4) Signal Constellations: An 8-PAM signal constellation is shown in the figure below. Assume the signals are used with equal probability on an AWGN channel with two-sided noise spectral density $N_o/2$ watts/Hz, and that the receiver makes minimum distance decisions.



a) 3 marks: Determine and sketch the decision boundaries Voronoi regions.

b) 5 marks: Find the *exact* probability of *symbol* error in terms of the *bit* energy-to-noise ratio E_b/N_o. N.B. Note the italics.

c) 2 marks: Sketch another 8-PAM signal constellation that will have a smaller probability of error for the same E_b/N_o, and state why the error probability will be smaller.

5) Signal Constellatons Consider the set of signals $\{s_i(t)\}_{i=1}^{M}$, where the $s_i(t)$ have duration T. The signals have vector representations

$$\mathbf{s}_i = [s_{i1},\ s_{i2},\ \ldots,\ s_{iN}], \quad i = 1,\ 2,\ \ldots, M$$

where

$$s_{ij} = \begin{cases} +d \\ \text{or} \\ -d \end{cases} \quad \text{for all } i, j$$

a) 3 marks: Sketch this signal set in two ($N = 2$) and three ($N = 3$) dimensions.

b) 3 marks: How many signal vectors are there in N dimensions?

c) 4 marks: Specify a set of time-domain waveforms that correspond to these signal vectors. *Note: There is no unique solution.*