EE4601
Communication Systems

Week 12

Noncoherent Detection

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Noncoherent Detection

With noncoherent detection, the carrier phase is not recovered at the receiver. Information is transmitted in amplitude and frequency only, not in the phase of the carrier.

Transmit:

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t) u_T(t)$$

Receive:

$$r(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_c t + \Phi) u_T(t) + n(t)$$

Assume: Φ is a random variable with *pdf*

$$f_\Phi(\phi) = \begin{cases} \frac{1}{2\pi} & -\pi \leq \phi \leq \pi \\ 0 & \text{else} \end{cases}$$

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Noncoherent Detection

Note that f_c is usually very large.

$$c = f_c \lambda_c \rightarrow \lambda_c = \frac{c}{f_c}$$

At 1 GHz, $\lambda_c \approx = 0.3$ m

Therefore, a change in radio path length of 30 cm causes a 2π change in carrier phase.

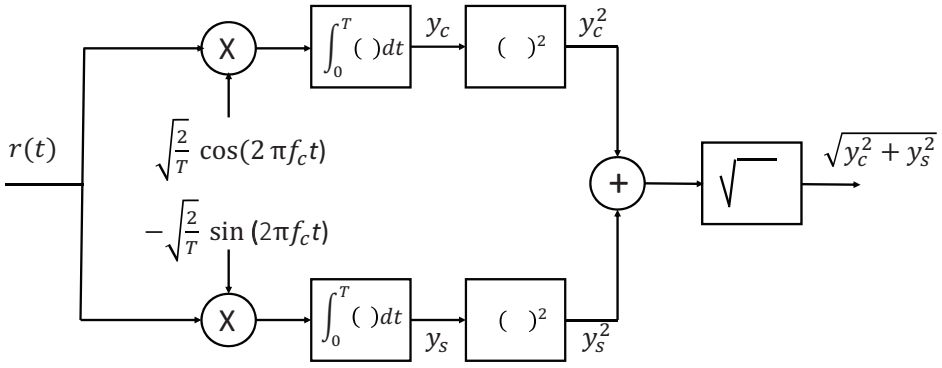
A vehicle (aircraft) travelling 1000 km/h will travel 278 m/s and experience a carrier phase change of $278/0.3 \times 2\pi = 1853\pi$ rad/s.

Rather than attempting to track this rapidly varying carrier phase, a noncoherent receiver may be used instead that does not require the carrier phase.

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Square-Law Correlation Detector

$$r(t) = \sqrt{\frac{2E}{T}} \cos\Phi \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\Phi \sin(2\pi f_c t) + n(t)$$



In the absence of noise $n(t)$,

$$y_c = \sqrt{E}\cos\Phi, \;\; y_s = \sqrt{E}\sin\Phi$$

$$y_c^2 + y_s^2 = E \quad \text{the energy in the waveform}$$

The above receiver is called a *square-law correlation detector.*

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Noncoherent BFSK

Information can only be transmitted in amplitude and frequency. Consider BFSK waveforms where

$$\begin{aligned} s_1(t) &= \sqrt{\frac{2E}{T}}\cos(2\pi f_c t)u_T(t) \\ s_2(t) &= \sqrt{\frac{2E}{T}}\cos(2\pi(f_c+\Delta f)t)u_T(t) \end{aligned}$$

Suppose these waveforms are transmitted on an ideal channel with a propagation delay t_d. Then the noiseless received waveforms (on the interval $[0,T]$) are

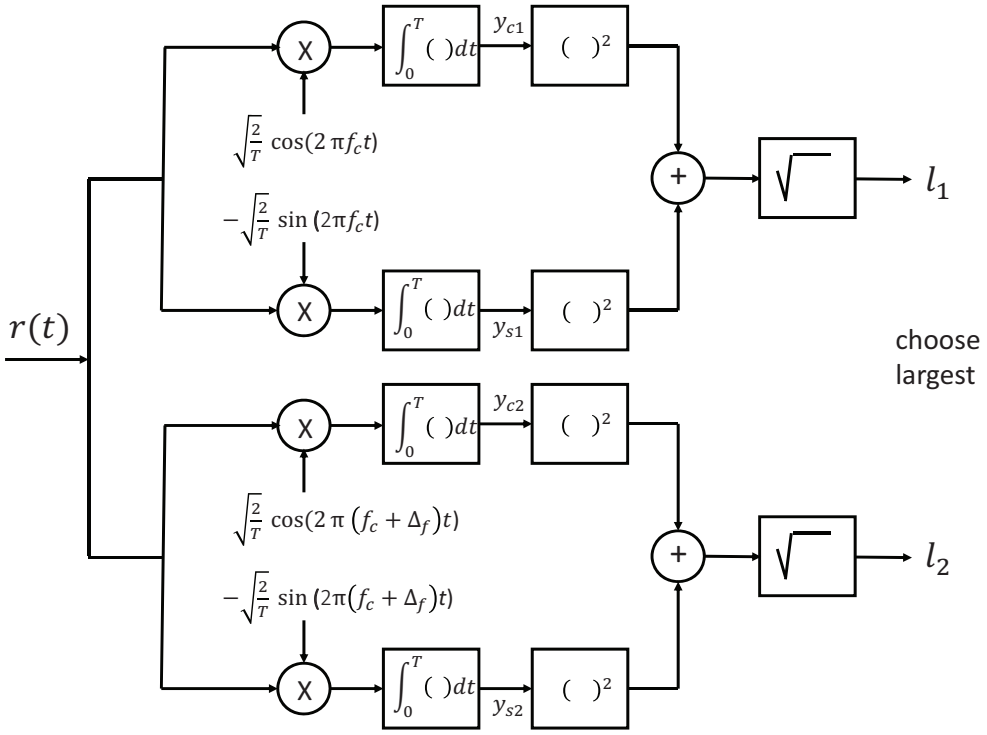
$$\hat{s}_1(t) = \sqrt{\frac{2E}{T}}\cos(2\pi f_c(t-t_d))u_T(t) = \sqrt{\frac{2E}{T}}\cos(2\pi f_c t+\Phi_1)u_T(t)$$

$$\hat{s}_2(t) = \sqrt{\frac{2E}{T}}\cos(2\pi(f_c+\Delta f)(t-t_d))u_T(t) = \sqrt{\frac{2E}{T}}\cos(2\pi(f_c+\Delta f)t+\Phi_2)u_T(t)$$

where $\Phi_1 = -2\pi f_c t_d$ and $\Phi_2 = -2\pi(f_c+\Delta f)t_d)$. If $\Delta f = \frac{1}{T}$, then the received waveforms $\hat{s}_1(t)$ and $\hat{s}_2(t)$ are orthogonal for arbitrary Φ_1 and Φ_2.

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Noncoherent BFSK



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Error Probability of Noncoherent BFSK

Assume $s_1(t)$ was sent. The probability of bit error is

$$P_b = P(l_2 > l_1)$$

Finding P_b requires the conditional pdfs of l_1 and l_2 given that $s_1(t)$ was sent.

$$\begin{aligned} y_{c1} &= \int_0^T r(t)\sqrt{\frac{2}{T}}\cos(2\pi f_c t)dt \\ &= \int_0^T \left(\sqrt{\frac{2E}{T}}\cos(2\pi f_c t + \Phi_1) + n(t)\right)\sqrt{\frac{2}{T}}\cos(2\pi f_c t)dt \\ &= \sqrt{E}\cos\Phi_1 + n_{c1} \end{aligned}$$

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Error Probability of Noncoherent BFSK

$$\begin{aligned} y_{s1} &= \int_0^T r(t)\sqrt{\frac{2}{T}}\sin(2\pi f_c t)dt \\ &= \int_0^T \left(\sqrt{\frac{2E}{T}}\cos(2\pi f_c t + \Phi_1) + n(t)\right)\sqrt{\frac{2}{T}}\sin(2\pi f_c t)dt \\ &= \sqrt{E}\sin\Phi_1 + n_{s1} \end{aligned}$$

The detector output is

$$l_1 = \sqrt{y_{c1}^2 + y_{s1}^2}$$

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Error Probability of Noncoherent BFSK

Likewise,

$$\begin{aligned} y_{c2} &= \int_0^T \left(\sqrt{\frac{2E}{T}} \cos(2\pi f_c t + \Phi_1) + n(t) \right) \sqrt{\frac{2}{T}} \cos(2\pi(f_c + \Delta_f)t) dt \\ &= n_{c2} \\ y_{s2} &= \int_0^T \left(\sqrt{\frac{2E}{T}} \cos(2\pi f_c t + \Phi_1) + n(t) \right) \sqrt{\frac{2}{T}} \sin(2\pi(f_c + \Delta_f)t) dt \\ &= n_{s2} \end{aligned}$$

The first term in each integral is zero because $s_1(t)$ and $s_2(t)$ are orthogonal for arbitary Φ_1 and Φ_2.

The detector output is

$$l_2 = \sqrt{y_{c2}^2 + y_{s2}^2} = \sqrt{n_{c2}^2 + n_{s2}^2}$$

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Error Probability of Noncoherent BFSK

$$n_{c1} = \int_0^T n(t)\sqrt{\frac{2}{T}}\cos(2\pi f_c t)dt$$
$$n_{s1} = \int_0^T n(t)\sqrt{\frac{2}{T}}\sin(2\pi f_c t)dt$$

n_{c1} and n_{s1} are Gaussian random variables with

$$\mathrm{E}[n_{c1}] = \mathrm{E}[n_{s1}] = 0,\ \sigma^2_{n_{c1}} = \sigma^2_{n_{s1}} = \frac{N_0}{2},\ \mathrm{E}[n_{c1}n_{s1}] = 0$$

The fact that that $\mathrm{E}[n_{c1}n_{s1}] = 0$ follows from the property

$$\frac{2}{T}\int_0^T \cos(2\pi f_c t)\sin(2\pi f_c t)dt = 0$$

for $f_c T \gg 1$.

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Error Probability of Noncoherent BFSK

Likewise

$$n_{c2} = \int_0^T n(t)\sqrt{\frac{2}{T}}\cos(2\pi(f_c+\Delta_f)t)dt$$
$$n_{s2} = \int_0^T n(t)\sqrt{\frac{2}{T}}\sin(2\pi(f_c+\Delta_f)t)dt$$

n_{c2} and n_{s2} are Gaussian random variables with

$$\mathrm{E}[n_{c2}] = \mathrm{E}[n_{s2}] = 0,\ \sigma^2_{n_{c2}} = \sigma^2_{n_{s2}} = \frac{N_0}{2},\ \mathrm{E}[n_{c2}n_{s2}] = 0$$

Since $s_1(t)$ and $s_2(t)$ are orthogonal, it also follows that

$$\mathrm{E}[n_{c1}n_{c2}] = 0, \mathrm{E}[n_{c1}n_{s2}] = 0, \mathrm{E}[n_{s1}n_{c2}] = 0, \mathrm{E}[n_{s1}n_{s2}] = 0$$

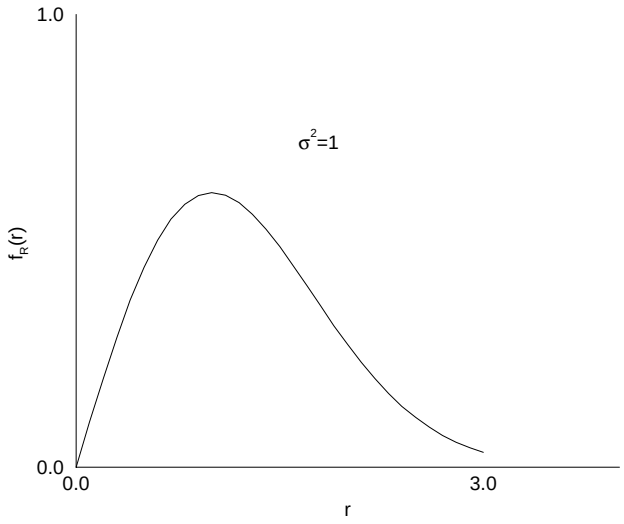
Hence, n_{c1}, n_{s1}, n_{c2}, n_{s2} are i.i.d. Gaussian $\sim N(0, N_0/2)$.

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Density Function of l_2

l_2 has a Rayleigh distribution and, for our problem,

$$f_{l_2}(x) = \frac{2x}{N_0} \exp\left\{-\frac{x^2}{N_0}\right\} \quad , x \geq 0$$

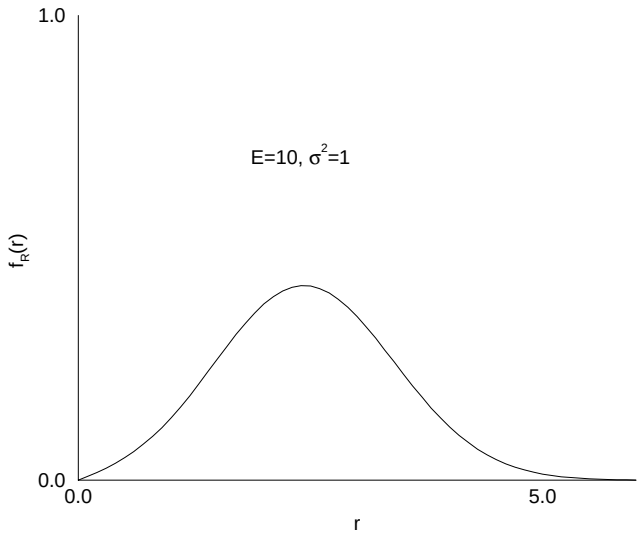


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Density Function of l_1

l_1 has a Rice distribution, and for our problem

$$f_{l_1}(x) = \frac{2x}{N_o} \exp\left\{-\frac{x^2+E}{N_o}\right\} I_0\left(\frac{2\sqrt{E}x}{N_o}\right) \quad , x \geq 0$$



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Bit Error Probability

$$\begin{aligned} P_b &= P(l_2 > l_1 | s_1(t) \text{ sent}) \\ &= \int_0^\infty P(l_2 > l_1 | l_1 \text{ and } s_1(t) \text{ sent}) f(l_1) dl_1 \end{aligned}$$

However,

$$P(l_2 > l_1 | l_1 \text{ and } s_1(t) \text{ sent}) = \int_{l_1}^\infty f(l_2) dl_2 = \exp\left\{-\frac{l_1^2}{N_0}\right\}$$

Hence,

$$\begin{aligned} P_b &= \int_0^\infty \exp\left\{-\frac{l_1^2}{N_0}\right\} \frac{2l_1}{N_0} \exp\left\{-\frac{l_1^2 + E}{N_0}\right\} I_0\left(\frac{2l_1\sqrt{E}}{N_0}\right) dl_1 \\ &= \int_0^\infty \frac{2l_1}{N_0} \exp\left\{-\frac{2l_1^2 + E}{N_0}\right\} I_0\left(\frac{2l_1\sqrt{E}}{N_0}\right) dl_1 \end{aligned}$$

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Error Probability

Define $v = \frac{2l_1}{\sqrt{N_0}}$. Then,

$$P_b = \frac{1}{2}\exp\left\{-\frac{E}{2N_0}\right\}\int_0^\infty v\exp\left\{-\frac{v^2+a^2}{2}\right\}I_0\left(av\right)dv$$

where $a = \sqrt{\frac{E}{N_0}}$.

However, the integral is a Rice pdf that is being integrated over its entire range, i.e., the integral is equal to 1.

Therefore,

$$P_b \;=\; \frac{1}{2}\exp\left\{-\frac{E}{2N_0}\right\} = \frac{1}{2}\exp\left\{-\frac{E_b}{2N_0}\right\}$$

since $E_b = E$.

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