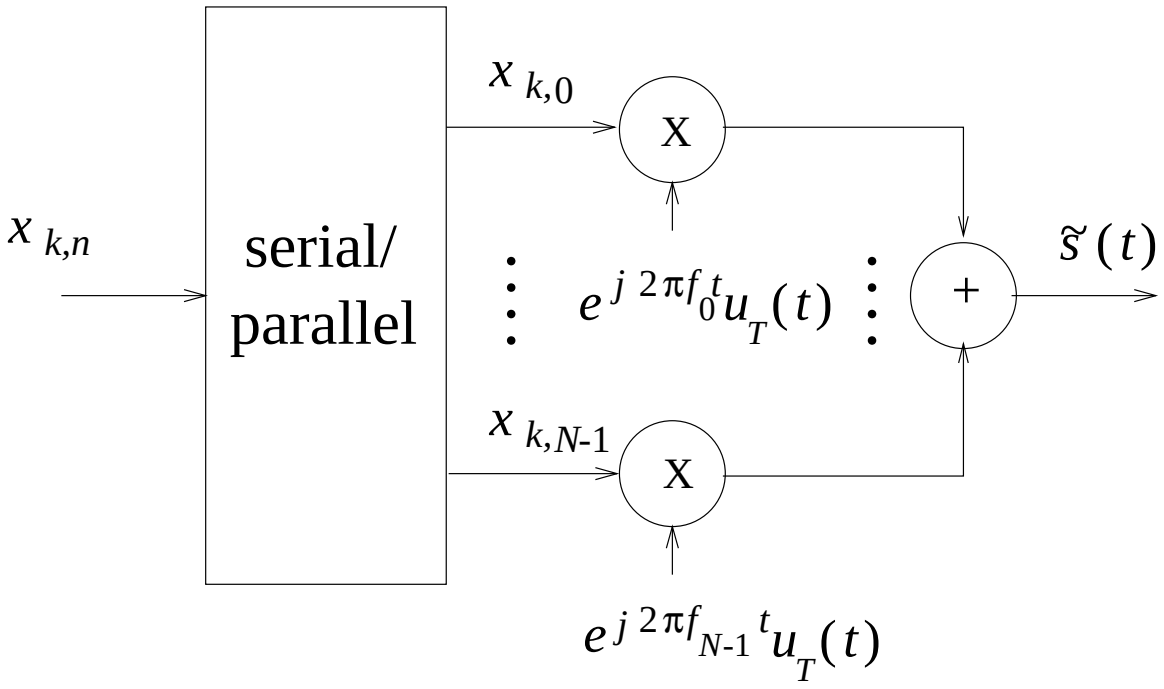
ECE4601
Communication Systems

Week 13

Orthogonal Frequency Division Multiplexing

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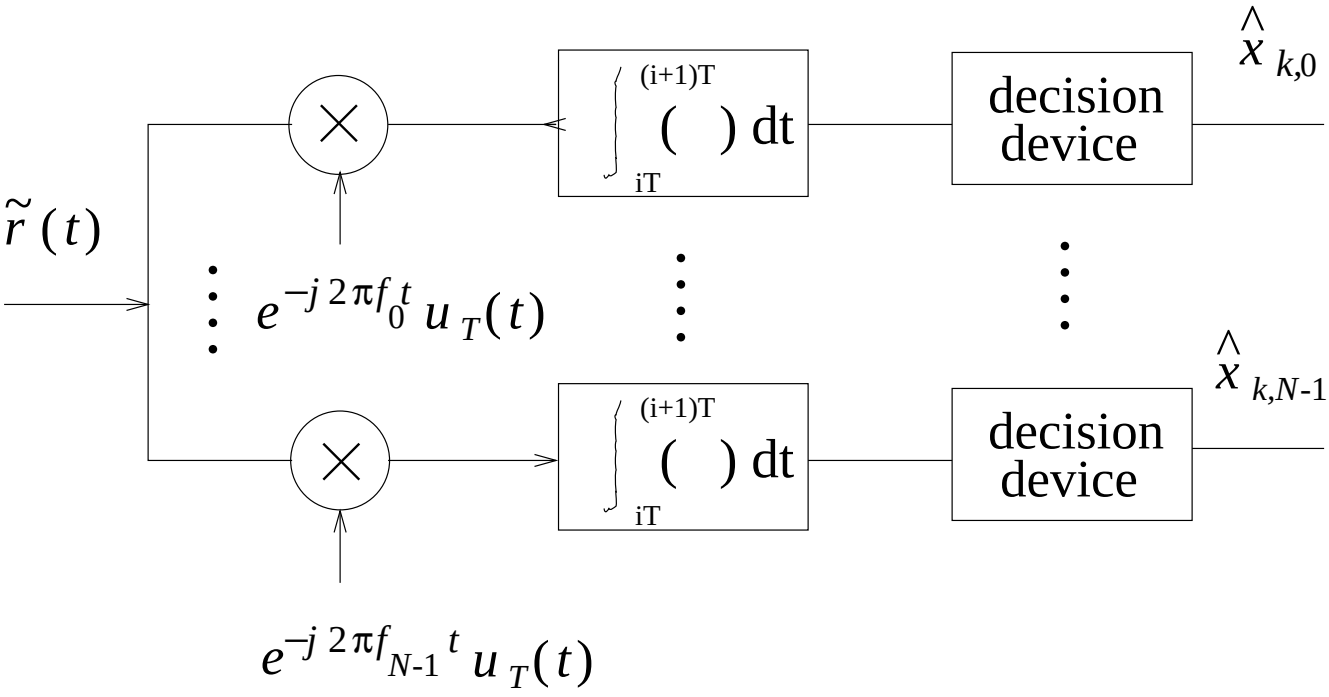
Block Modulation



Block transmission through the principle of multi-carrier modulation

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Block Demodulation



$$x_{i,n} = \frac{1}{T}\int_{iT}^{(i+1)T} r(t)\exp\{-j2\pi f_n t\}dt$$

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OFDM Signal Description

- The OFDM complex envelope and bandpass signal are

$$\begin{aligned} \tilde{s}(t) &= A \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} x_{k,n} \exp\left\{\frac{j2\pi nt}{NT_s}\right\} u_T(t-kT), \quad T = NT_s \\ s(t) &= \mathrm{Re}\left[\tilde{s}(t)e^{j2\pi f_c t}\right] \end{aligned}$$

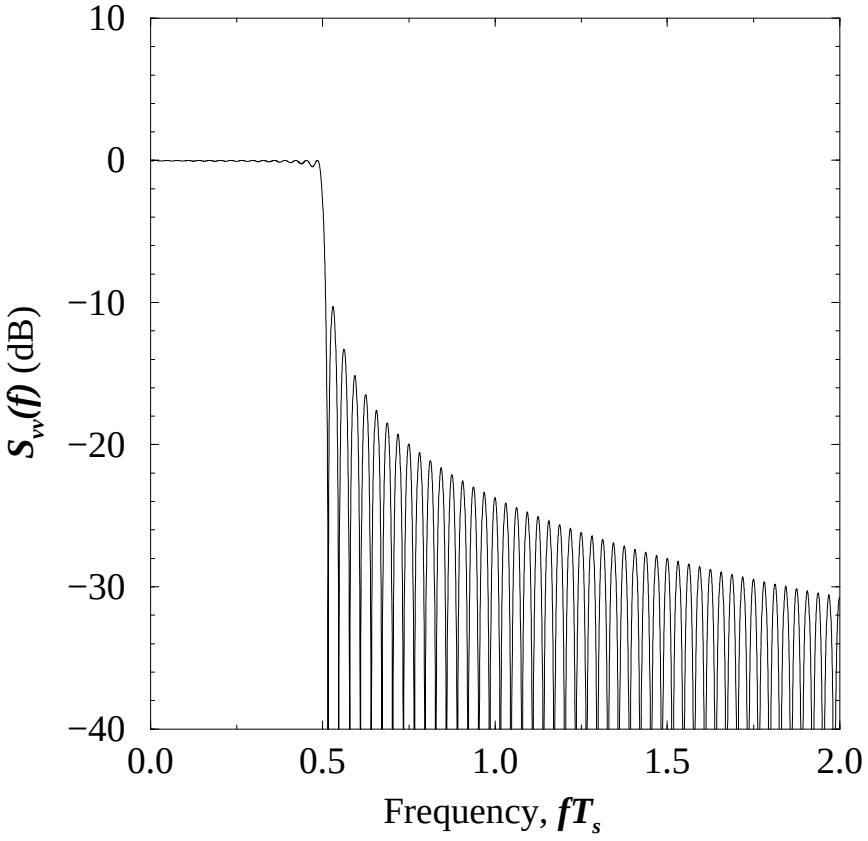
  where

$$\begin{aligned} k &= \text{block index} \\ x_{k,n} &= x_{k,n}^I + jx_{k,n}^Q = \text{complex data symbol on } n\text{th subcarrier in block } n \\ N &= \text{block size, usually a power of 2} \\ u_T(t) &= u(t) - u(t-T) = \text{unit amplitude length-}T \text{ rectangular pulse} \\ \Delta_f &= 1/T \end{aligned}$$

- The sub-carrier frequency separation of $1/T$ ensures that the sub-carriers are orthogonal in time regardless of the random sub-carrier phase that is introduced by the data symbols.

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Power Spectrum of OFDM



Power spectral density of OFDM with $N = 32$ sub-carriers, $T = NT_s$.

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Practical Implementation

- Considering the data block at epoch 0, the OFDM signal has the complex envelope (here we suppress the block index 0)

$$\begin{aligned} \tilde{s}(t) &= A\sum_{n=0}^{N-1} x_n \exp\left\{\frac{j2\pi nt}{NT_s}\right\} u_T(t), \quad T = NT_s \\ s(t) &= \mathrm{Re}\left[\tilde{s}(t)e^{j2\pi f_c t}\right] \end{aligned}$$

- Sample the complex envelope at $t = kT_s$ to yield the sequence

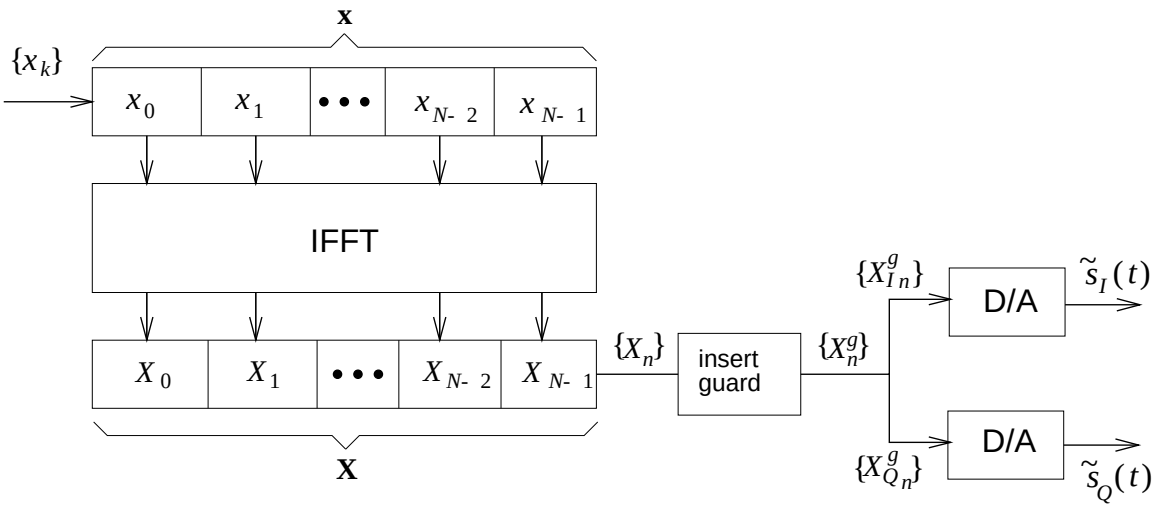
$$X_k = \tilde{s}(kT_s) = A\sum_{n=0}^{N-1} x_n \exp\left\{\frac{j2\pi nk}{N}\right\}, \quad k = 0,\ 1,\ \ldots,\ N-1$$

  - The sampled envelope sequence has duration $T = NT_s$
  - The vector $\mathbf{X} = \{X_n\}_{n=0}^{N-1}$ is just the inverse DFT (IFFT) of the vector $A\mathbf{x} = A\{x_k\}_{k=0}^{N-1}$.

- The OFDM modulator is just an IFFT circuit followed by a D/A converter.

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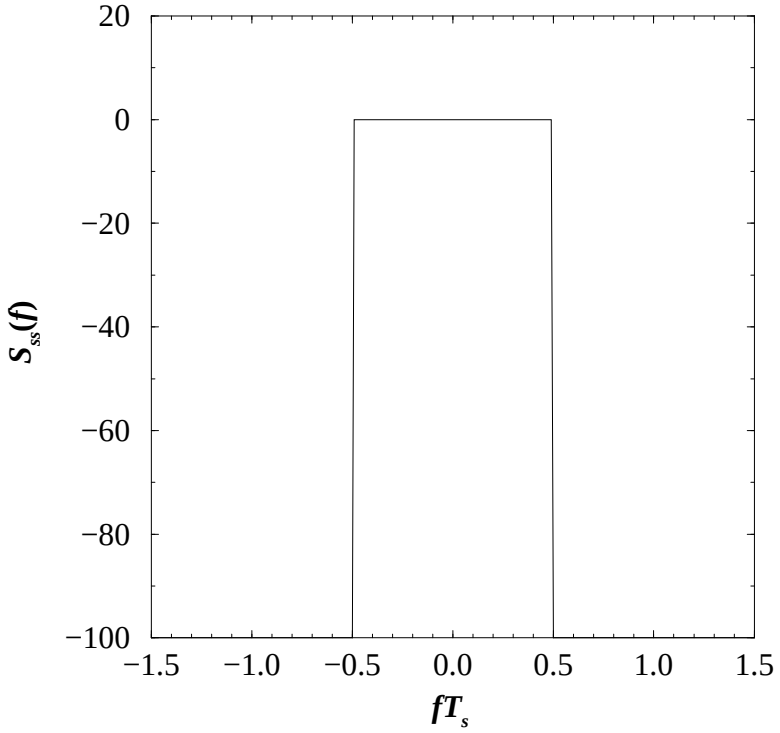
OFDM Transmitter



FFT-based OFDM Transmitter

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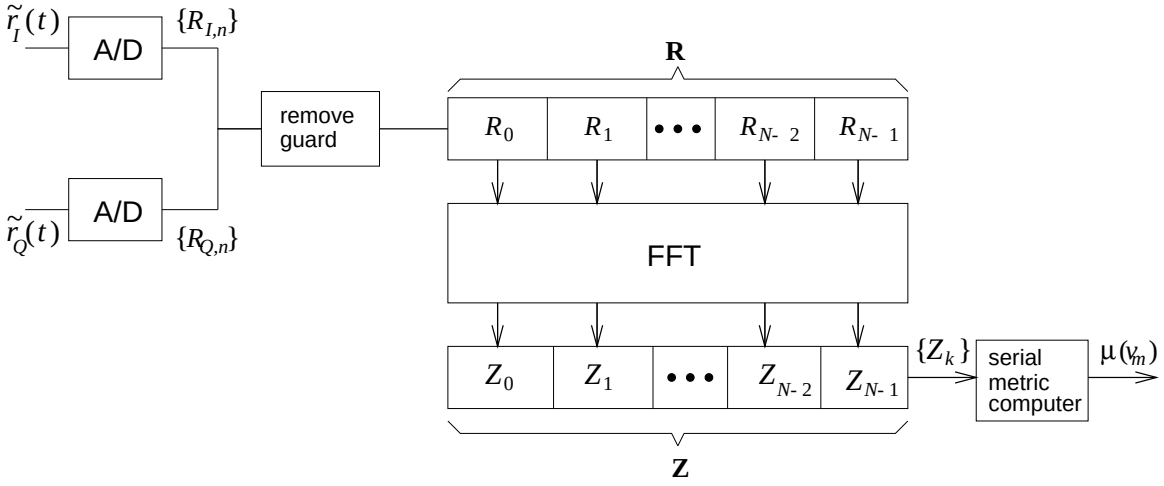
OFDM Spectrum with Ideal D/A



Psd of OFDM with implemented with IFFT algorithm.

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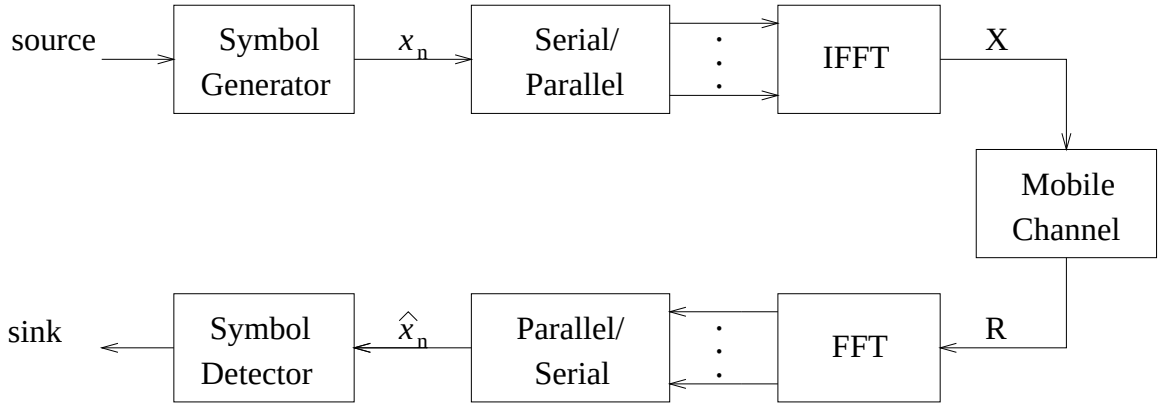
OFDM Receiver



FFT-based OFDM Receiver

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OFDM System



OFDM System

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Combating ISI with OFDM

- To prevent intersymbol interference (ISI) between OFDM blocks a guard interval is inserted between the blocks.
- A very convenient way to do this is to append a **cyclic prefix** or a **cyclic suffix** to the IFFT coefficients.
- The IFFT output sequence with the addition of cyclic suffix guard interval is

$$X_n^g = X_{(n)_N}, \quad n = 0,\ 1,\ \ldots,\ N + G - 1$$

$$\begin{aligned} G &= \text{length of the guard interval} \\ (n)_N &= \text{residue of } n \text{ modulo } N \end{aligned}$$

- The transmission rate with guard interval is $\frac{1}{T_s^g} = \frac{1}{T_s}(1 + \frac{G}{N})$, where $1/T_s$ is the source symbol rate.

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Circular Convolution

- Suppose that the sequence $\{X_k^g\}_{n=0}^{N-G-1}$ is transmitted over a channel with discrete-time impulse response $g_m, m = 0, \ldots, L$.
- The received sequence is

$$R_n^g = \sum_{m=0}^{L} g_m X_{n-m}^g$$

- The receiver first removes the guard interval according to

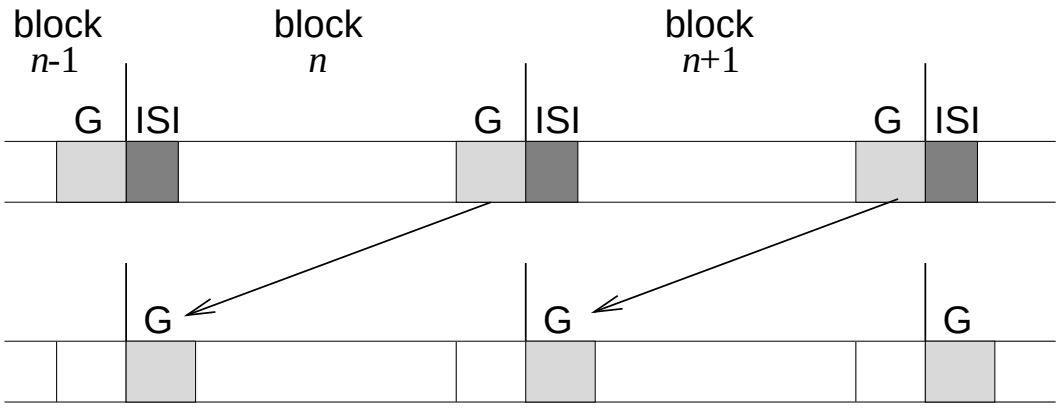
$$R_n = R_{G+(n-G)_N}^g, \quad 0 \leq n \leq N-1$$

- If $G \geq L$, then the received samples after the removal of the guard interval are

$$R_n = \sum_{m=0}^{L} g_m X_{(n-m)_N}, \quad 0 \leq n \leq N-1$$

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Cyclic Suffix



Removal of ISI using a cyclic suffix

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Removal of ISI

- The OFDM demodulator then performs an FFT on the vector $\mathbf{R} = \{R_n\}_{n=0}^{N-1}$. The demodulated sequence is

$$\begin{aligned} Z_i &= \frac{1}{N}\sum_{n=0}^{N-1} R_n e^{-j\frac{2\pi ni}{N}} \\ &= \eta_i A x_i \ , \quad 0 \le i \le N-1 \end{aligned}$$

  where

$$\eta_i = \sum_{m=0}^{L} g_m e^{-j\frac{2\pi mi}{N}}$$

- Notice that Z_i is equal to Ax_i multiplied by the equivalent complex channel gain η_i.
- The ISI due to the channel has been completely removed.

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