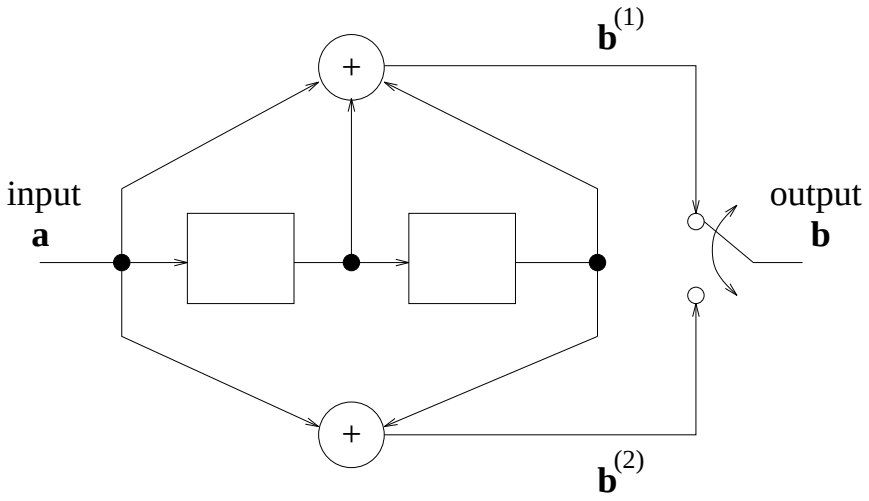
ECE4601
Communication Systems

Week 15

Convolutional Codes

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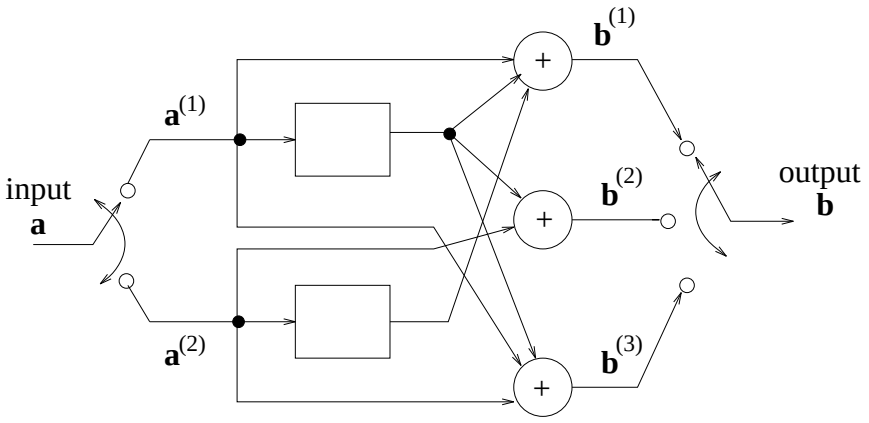
Rate-1/2 Encoder



Rate 1/2 binary convolutional encoder.

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Rate-2/3 Encoder



Rate 2/3 binary convolutional encoder.

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Generator Sequences

- The connections to the shift register are described by *generator sequences*.
  - Rate 1/2 code example

$$\begin{aligned} \mathbf{g}^{(1)} &= (1,\ 1,\ 1) \\ \mathbf{g}^{(2)} &= (1,\ 0,\ 1) \end{aligned}$$

  - Rate 2/3 code example

$$\begin{aligned} \mathbf{g}_1^{(1)} &= (1,\ 1), \quad \mathbf{g}_1^{(2)} = (0,\ 1), \quad \mathbf{g}_1^{(3)} = (1,\ 1) \\ \mathbf{g}_2^{(1)} &= (0,\ 1), \quad \mathbf{g}_2^{(2)} = (1,\ 0), \quad \mathbf{g}_2^{(3)} = (1,\ 0)\ . \end{aligned}$$

- The jth output corresponding to the ith input sequence $\mathbf{a}^{(i)}$, $\mathbf{b}_i^{(j)}$, is

$$\mathbf{b}_i^{(j)} = \mathbf{a}^{(i)} * \mathbf{g}_i^{(j)}$$

  and

$$\mathbf{b}^{(j)} = \sum_i \mathbf{b}_i^{(j)}$$

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Finite State Machine

- The encoder is a finite state machine
- The state of the encoder at time ℓ is

$$\varrho_\ell = \left(a_{\ell-1}^{(1)}, \cdots, a_{\ell-\nu_1}^{(1)} \; ; \cdots \; ; a_{\ell-1}^{(k)}, \cdots, a_{\ell-\nu_m}^{(k)}\right)$$

  where ν_i is the length of the shift register for input i.

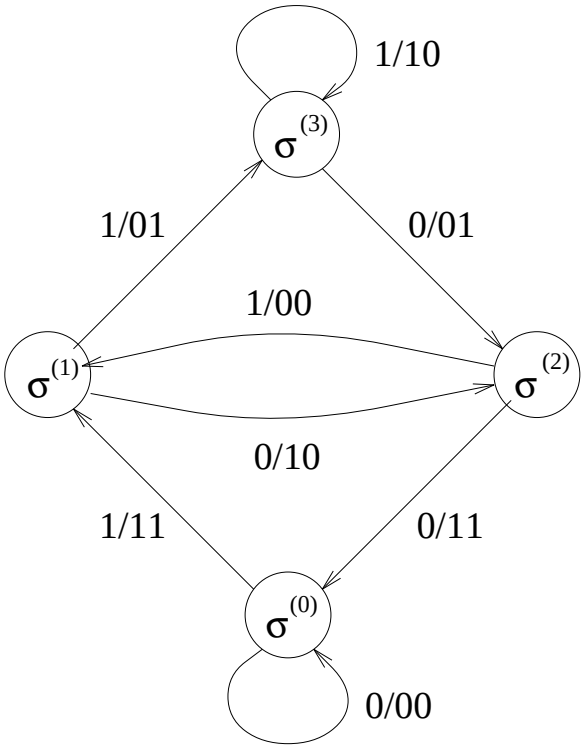
  – The state of the encoder is defined by the memory contents in the various shift registers of the encoder

- There are a total of $N_S = 2^{\nu_T}$ encoder states, where $\nu_T \stackrel{\Delta}{=} \sum_{i=1}^{k} \nu_i$ is defined as the **total encoder memory**.
- For a rate-$1/n$ code, the encoder state at epoch ℓ is simply

$$\varrho_\ell = (a_{\ell-1}, \cdots, a_{\ell-\nu})$$

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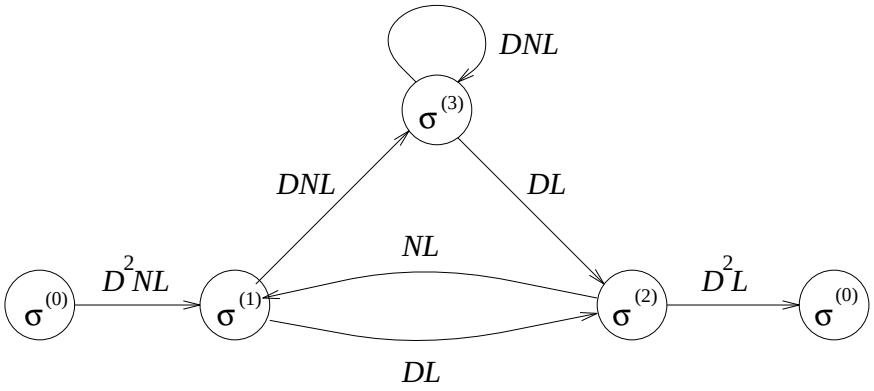
Rate-1/2 code



State diagram for the rate-1/2 binary convolutional encoder example.

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Augmented State Diagram



Modified state diagram for the rate-1/2 binary convolutional encoder.

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Transfer Function

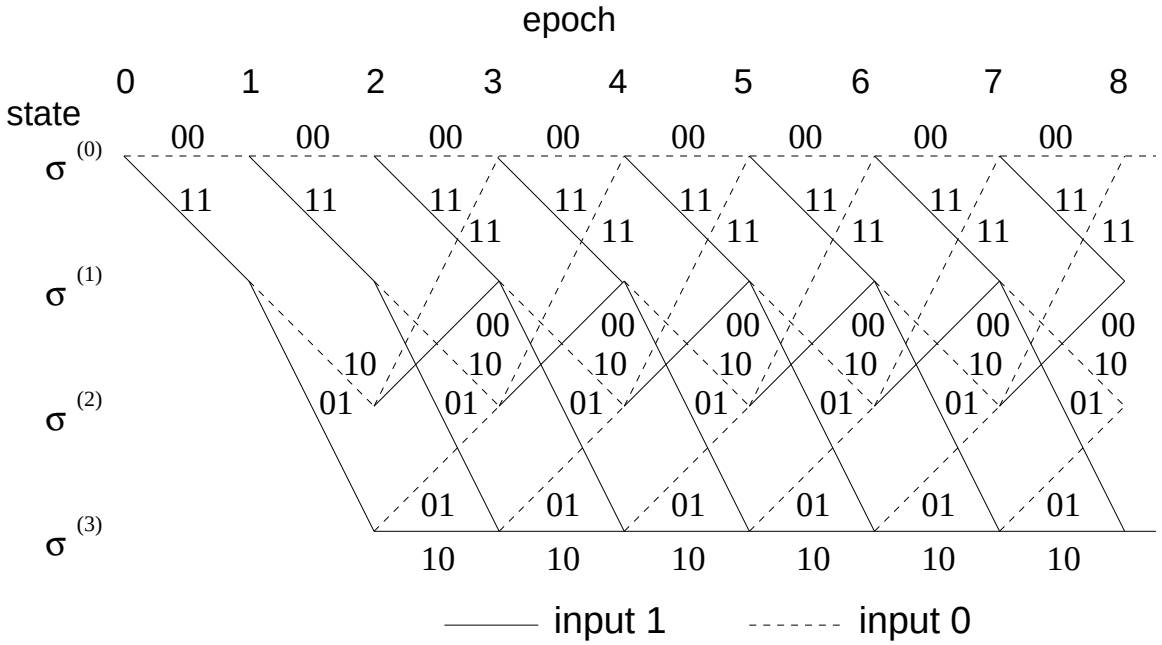
- Any appropriate technique can be used to obtain the transfer function, such as Mason's formula
- For the rate-1/2 code example

$$\begin{aligned} T(D,N,L) &= \frac{D^5NL^3}{1-DNL(L+1)} \\ &= D^5L^3N + D^6N^2L^4(L+1) + D^7N^3L^5(L+1)^2 \\ &\quad + \cdots + D^{k+5}N^{k+1}L^{k+3}(L+1)^k + \cdots \end{aligned}$$

- The term $D^{k+5}N^{k+1}L^{k+3}(L+1)^k$ means there are 2^k paths at Hamming distance $k+5$ from the all-zeroes path, caused by $k+1$ input ones. Of these 2^k paths, $\binom{k}{n}$ have length $k+n+3$.
- Free Hamming distance $d_{\text{free}} = 5$. This the smallest weight non-zero weight codeword.

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Trellis Diagram



Trellis diagram for the rate-1/2 binary convolutional code.

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