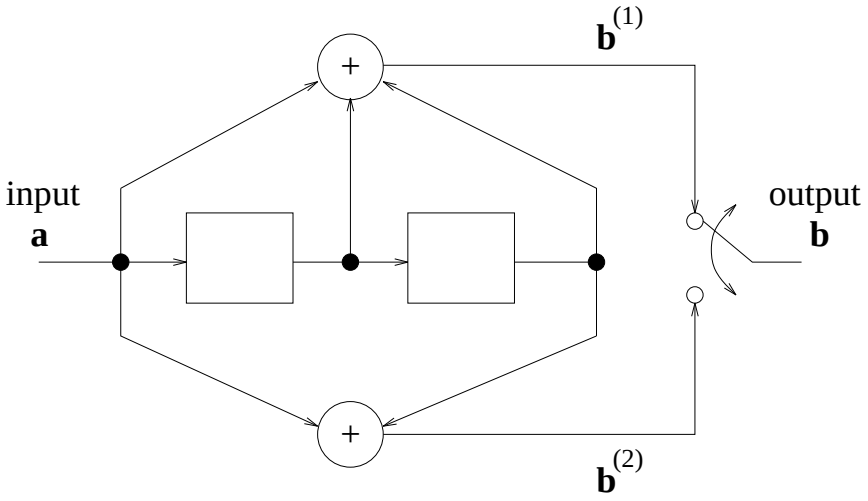
ECE4601
Communication Systems

Week 16

Viterbi Algorithm

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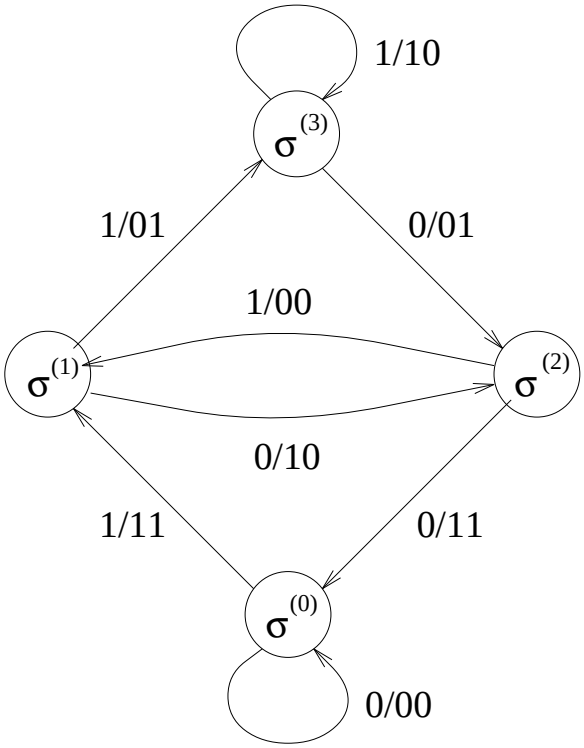
Rate-1/2 Encoder



Rate 1/2 binary convolutional encoder.

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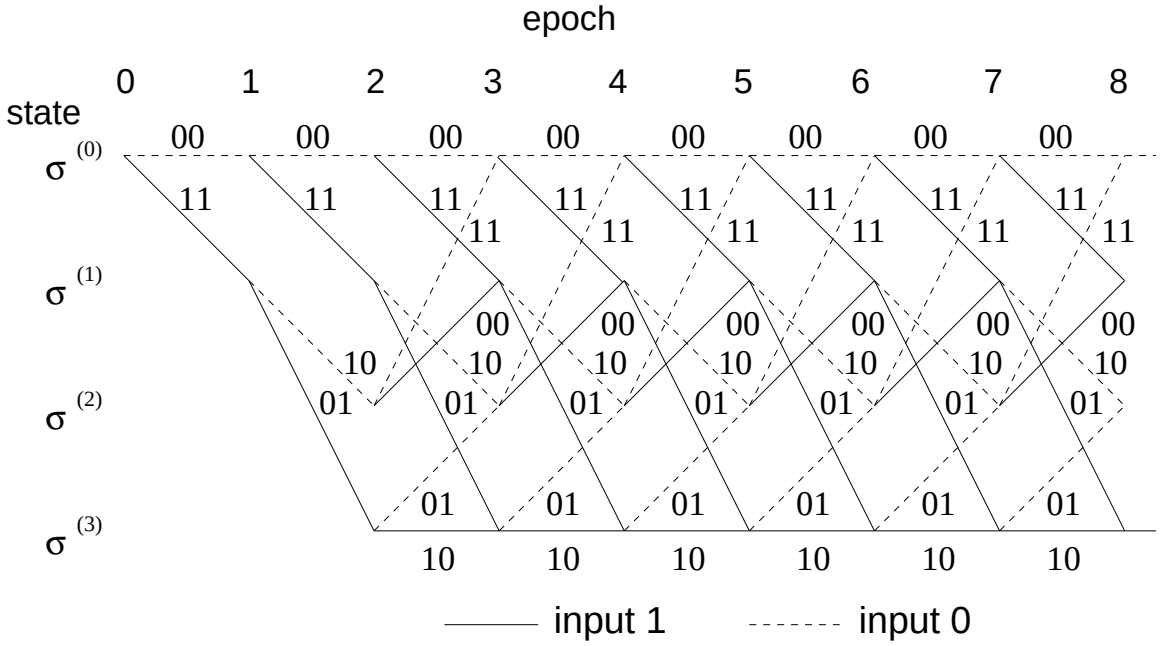
Rate-1/2 code



State diagram for the rate-1/2 binary convolutional encoder example.

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Trellis Diagram



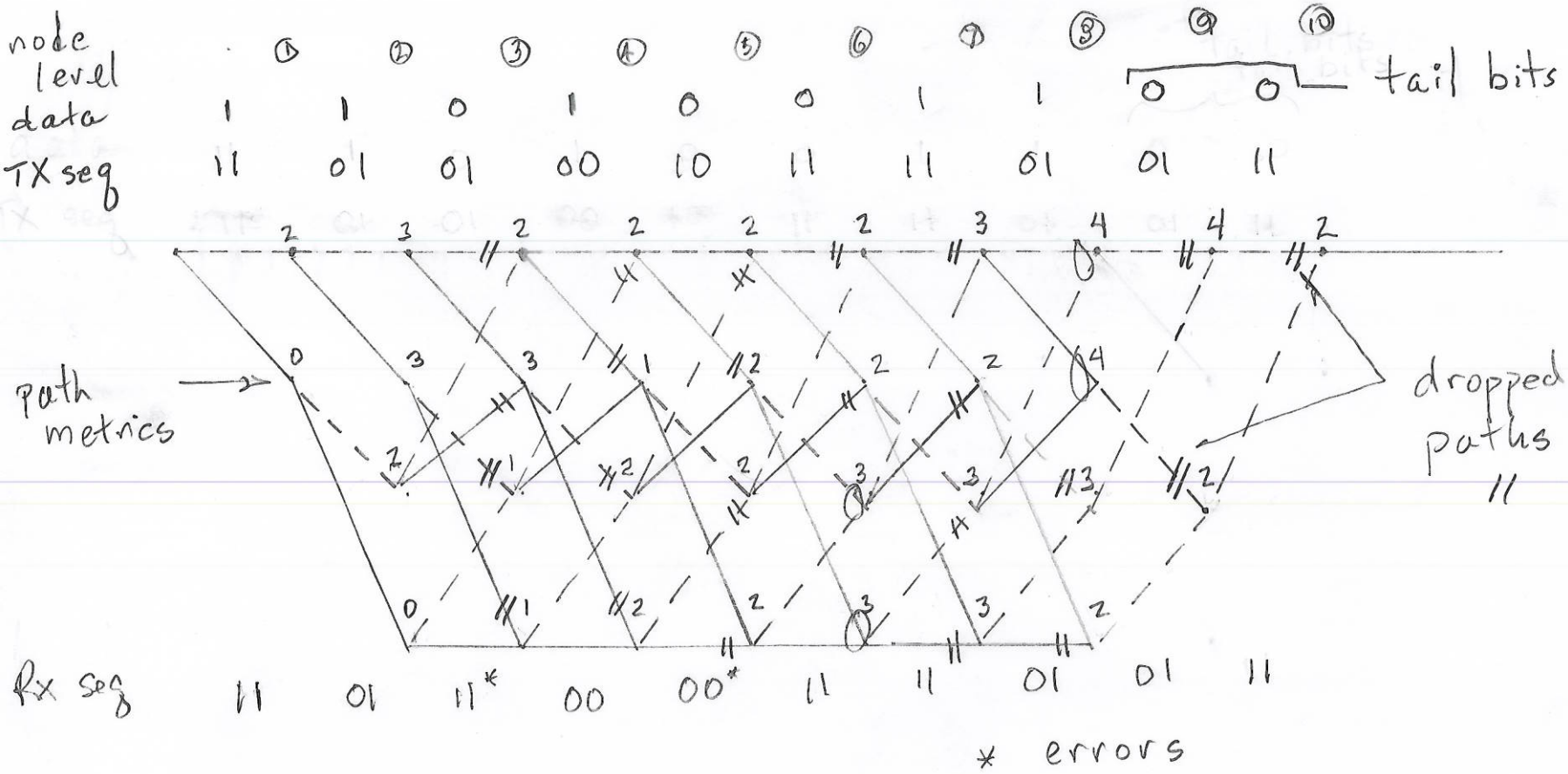
Trellis diagram for the rate-1/2 binary convolutional code.

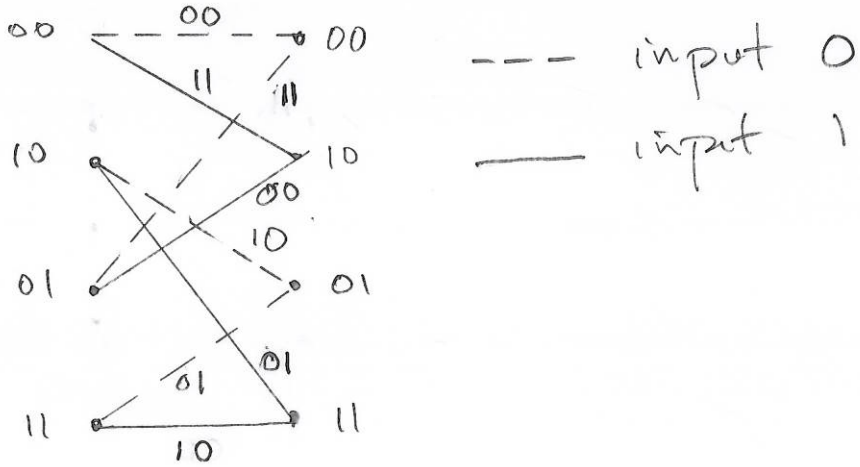
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Viterbi Algorithm

- The Viterbi algorithm searches through the N_S-state trellis for the most *likely* transmitted code symbol **sequence x**. This search process is called *maximum likelihood sequence estimation (MLSE).*
- Consider transmission of a coded sequence over a binary symmetric channel (BSC) with crossover probability p.
- The received sequence is $\mathbf{y} = \mathbf{x} + \mathbf{e}$, where $\mathbf{e}$ is the error sequence.
- For a BSC, the most likely transmitted sequence is the one that is closest to the received sequence $\mathbf{y}$ in Hamming distance, i.e., it differs from the received sequence in the fewest number of positions.
- The Viterbi algorithm provides a systematic method for finding this sequence.

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node level
data
TX seq
tail bits
path metrics
dropped paths
Rx seq
* errors


00
10
01
11
input 0
input 1

Viterbi Algorithm

- The branch metric at the kth epoch is defined as $\mu_k = d(\mathbf{y}_k, \mathbf{x}_k)$, where $\mathbf{y}_k$ is the sequence of received code bits and $\mathbf{x}_k$ is the *hypothesis sequence*.
  - For example, suppose the received sequence is $\mathbf{y}_k = (01)$. If the *hypothesis sequence* is $\mathbf{x}_k = (11)$, then $\mu_k = d(\mathbf{y}_k, \mathbf{x}_k) = 1$.
- The path metric is defined as

$$\Gamma(\varrho_k^{(i)}) = \sum_{\{k\}} \mu_k, \ i = 0, \cdots, N_S - 1$$

  where $\{\mu_k\}$ is the sequence of branch metrics along the surviving path $\check{\mathbf{x}}(\varrho_k^{(i)})$.
- At epoch k, assume that the algorithm has stored N_S **surviving sequences** $\check{\mathbf{x}}(\varrho_k^{(i)})$ (paths through the trellis - one for each state) along with their associated **path metrics** $\Gamma(\varrho_k^{(i)})$ (distances from the received sequence) that terminate at state $\varrho_k^{(i)}, \ i = 0, \cdots, N_S - 1$.

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Step-by-step

- Initialize $\Gamma(\varrho_0^{(i)}) = 0,\ i = 0, \cdots, N_S - 1$, i.e., initially set all path metrics to zero.
- After the vector $\mathbf{y}_k$ has been received, the Viterbi algorithm executes the following steps for each state $\varrho_{k+1}^{(j)}$, for $j = 0,\ \cdots,\ N_S - 1$:
  1. Compute the set of path metrics $\Gamma(\varrho_k^{(i)} \rightarrow \varrho_{k+1}^{(j)}) = \Gamma(\varrho_k^{(i)}) + \mu(\varrho_k^{(i)} \rightarrow \varrho_{k+1}^{(j)})$ for all possible paths through the trellis that terminate in state $\varrho_{k+1}^{(j)}$.
  2. Find $\Gamma(\varrho_{k+1}^{(j)}) = \max_i \Gamma(\varrho_k^{(i)} \rightarrow \varrho_{k+1}^{(j)})$ where, again, the maximization is over all possible paths through the trellis that terminate in state $\varrho_{k+1}^{(j)}$.
  3. Store $\Gamma(\varrho_{k+1}^{(j)})$ and its associated surviving sequence $\check{\mathbf{x}}(\varrho_{k+1}^{(j)})$. Drop all other paths.

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Some Notes

- In Step 1, $\mu(\varrho_k^{(i)} \to \varrho_{k+1}^{(j)})$ is the branch metric associated with the transition $\varrho_k^{(i)} \to \varrho_{k+1}^{(j)}$ and is computed by

$$\mu(\varrho_k^{(i)} \to \varrho_{k+1}^{(j)}) = d\left(\mathbf{y}_k, \mathbf{x}_k(\varrho_k^{(i)} \to \varrho_{k+1}^{(j)})\right)$$

  where $\mathbf{x}_k(\varrho_k^{(i)} \to \varrho_{k+1}^{(j)})$ is a symbol sequence that is uniquely determined by the state transition $\varrho_k^{(i)} \to \varrho_{k+1}^{(j)}$.

- After all states have been processed, the time index k is incremented and the whole algorithm repeats.

- With a "tail sequence" the encoder begins and ends in the all-zeroes state.

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