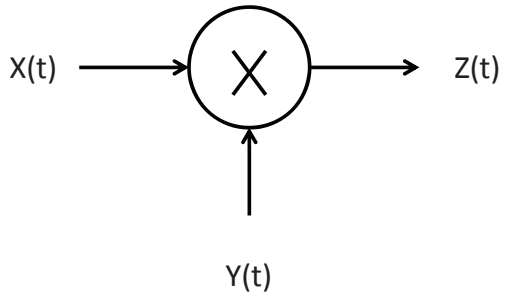
GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
Final Examination - Fall 2011
EE 4601: Communication Systems

Aids Allowed: Course textbook,
one $8\frac{1}{2} \times 11$ two-side crib sheet
Attempt all questions
All questions are of equal value

DATE: Monday December 12, 2011.
TIME: 8:00am - 10:50am
INSTRUCTOR: Professor G.L. Stüber

1) Random Processes: Suppose that two *statistically independent* random processes $X(t)$ and $Y(t)$ having autocorrelation functions $\phi_{XX}(\tau)$ and $\phi_{YY}(\tau)$, respectively, are input to a multiplier as shown in the the figure below



a) 4 marks: Find the autocorrelation function of the multiplier output $Z(t)$ in terms of the autocorrelation functions of $X(t)$ and $Y(t)$. *Hint: Start with the definition of the autocorrelation function that uses the expectation operator.*

b) 4 marks: Suppose that the autocorrelation functions of $X(t)$ and $Y(t)$ are given as follows:

$$\begin{aligned} \phi_{XX}(\tau) &= \frac{1}{T}\text{sinc}(\tau/T) \\ \phi_{YY}(\tau) &= \frac{2}{T}\text{sinc}(2\tau/T) \end{aligned}$$

Determine the power spectrum of the multiplier output $Z(t)$ and sketch it.

c) 2 marks: How much power is contained in the multiplier output $Z(t)$ in part b)?

2) Signal Space: Consider the follow two signal waveforms:

$$\begin{aligned} s_1(t) &= A\sin(2\pi f_c t)u_T(t) \\ s_2(t) &= A\sin(2\pi f_c t + \theta)u_T(t) \end{aligned}$$

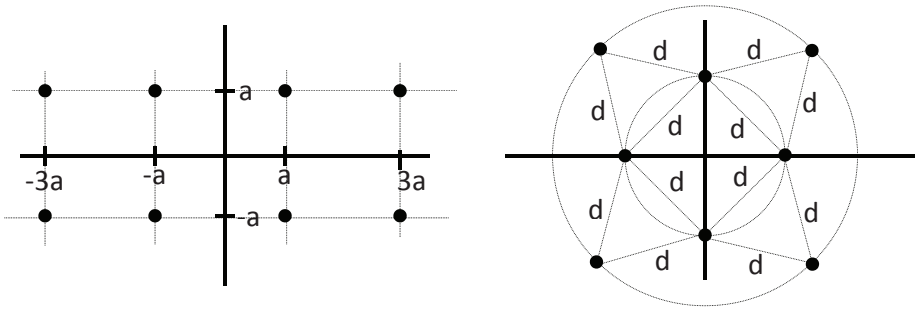
where $u_T(t) = u(t) - u(t-T)$.

a) 5 marks: Find a complete set of orthonormal basis functions for this set of signals and write down the corresponding signal vectors.

b) 4 marks: Suppose the waveforms are used for binary modulation on a additive white Gaussian noise channel with two-sided noise spectral density $N_o/2$ watts/Hz with coherent detection. What is the probability of bit error in terms of the angle θ?

c) 1 marks: What choice of θ will minimize the probability of bit error?

3) Signal Constellations: Consider the two different signal constellations shown below, where each constellation has 8 points.



a) 5 marks: Find the minimum distance $d_{\min}$ of each signal constellation in terms of the average symbol energy E_{av}.

b) 2 marks: Write down a simple upper bound on the probability of *symbol* error for each constellation in terms of the *bit* energy-to-noise ration E_b/N_o.

c) 3 marks: At high E_b/N_o, which constellation will be better in terms of error probability performance? How much better will it be in terms of the E_b/N_o (in dB) required to give the same error probability as the other constellation?

4) Pulse Shaping: A voice-band telephone channel passes the frequencies in the band from 300 to 3300 Hz. It is desired to design a modem that transmits at a symbol rate of 2400 symbols/sec, with the objective of achieving 9600 bits/sec.

a) 6 marks: Select an appropriate QAM constellation, carrier frequency, and the roll-off factor of a square-root raised cosine spectrum pulse.

b) 4 marks: Sketch the spectrum of the transmitted signal pulse and indicate important frequencies.

5) Coding: Suppose the generator matrix $\mathbf{G}$ for a (6,3) linear block code is given as follows:

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

a) 2 marks: What is the parity check matrix $\mathbf{H}$ for this code?

b) 4 marks: Generate the standard array for this code.

c) 2 marks: Calculate the syndrome vector for all the correctable error patterns.

d) 2 marks: Decode (i.e., find the message vector) the received sequence $\mathbf{y} = (101101)$.