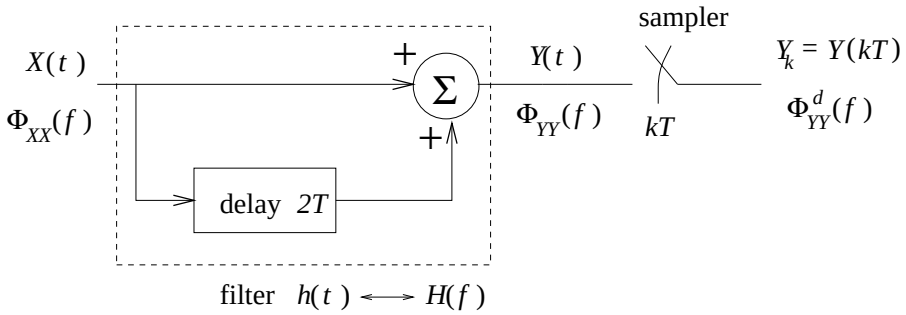
GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
Final Examination - Fall 2012
EE 4601: Communication Systems

Aids Allowed: Course textbook
one $8\frac{1}{2} \times 11$ two-side crib sheet
Attempt all questions
All questions are of equal value

DATE: Wednesday December 12, 2012.

TIME: 11:30am - 2:20pm
INSTRUCTOR: Professor G.L. Stüber

1) Random Processes: Consider the system shown below. The input $X(t)$ to the filter $h(t) \leftrightarrow H(f)$ is a wide-sense stationary random process with power density spectrum $\Phi_{XX}(f)$.



a) (**5 points**) Find the power spectral density $\Phi_{YY}(f)$ of the random process $Y(t)$ at the output of the filter $h(t)$ in terms of $\Phi_{XX}(f)$.

b) (**3 points**) What frequency components cannot be present in the filter output $Y(t)$?

c) (**2 points**) Find the power spectral density $\Phi^d_{YY}(f)$ of the discrete-time random process Y_k at the output of the sampler.

2) ISI Channels: Suppose that the frequency response characteristic of a channel can be approximated by

$$H(f) = 1 + \alpha \cos(2\pi f t_o) \quad ,$$

where $|\alpha| < 1$. A pulse $g(t)$ is passed through the channel.

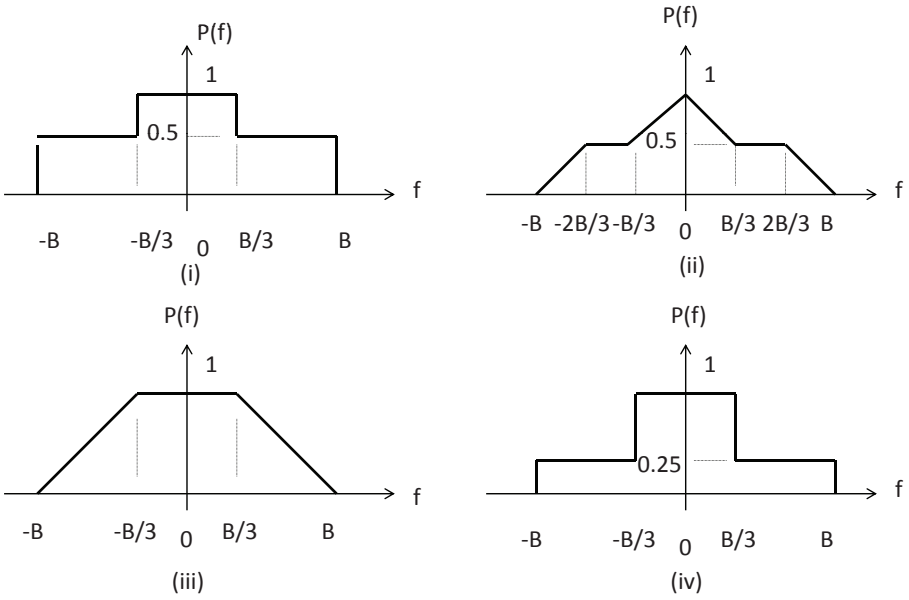
a) (**5 points**) Find the signal $y(t) = g(t) * h(t)$ at the output of the channel.

b) (**3 points**) Suppose the received signal $y(t)$ is passed through a filter matched to $g(t)$, such that $p(t) = g(t) * g(-t)$. Determine the sampled output of the matched filter $p_k = p(kT)$, at time $t = kT, k = 0, \pm 1, \pm 2, \ldots$.

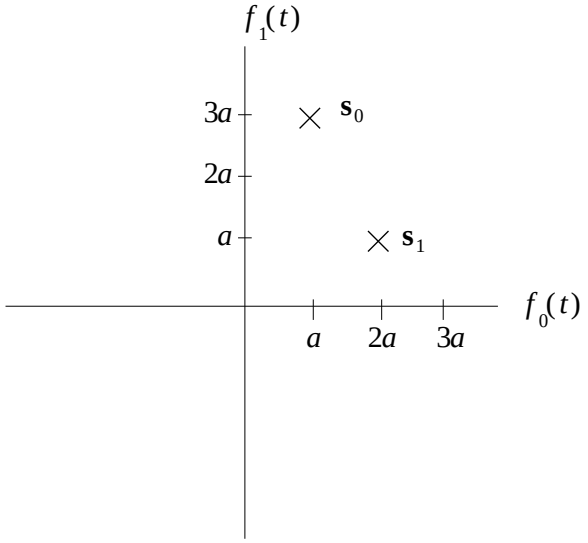
c) (**2 points**) What is the overall sampled pulse $p_k = p(kT)$ if $t_o = T$?

3) Intersymbol Interference: For each of the overall system responses in the figure below, $P(f) = G(f)C(f)H(f)$, where $G(f)$ is the transmit filter, $C(f)$ is the channel, $H(f)$ is the receiver filter.

a) (**10 points**) For each channel, determine if one can signal with zero ISI, and if so determine what the baud rate R is.



4) Signal Space: Consider a binary communication system whose signals are represented in the signal-space diagram shown below.



a) (**3 points**) Calculate the correlation coefficient ρ_{01} between the two signal waveforms $s_0(t)$ and $s_1(t)$.

b) (**3 points**) What is the average energy per information bit, E_b?

c) (**4 points**) Assuming an AWGN channel with a two-sided noise power spectral density of $N_o/2$ watts/Hz and minimum distance decisions, what is the probability of bit error in terms of E_b/N_o?

5) Error Probability: Consider the following set of four signal vectors.

$$\begin{aligned}
\mathbf{s}_1 &= \sqrt{E/6}\ (-1,-1,-1,+1,+1,+1) \\
\mathbf{s}_2 &= \sqrt{E/6}\ (-1,+1,-1,-1,+1,+1) \\
\mathbf{s}_3 &= \sqrt{E/6}\ (+1,-1,-1,+1,-1,-1) \\
\mathbf{s}_4 &= \sqrt{E/6}\ (+1,-1,+1,-1,-1,-1)
\end{aligned}$$

The messages are transmitted over an additive white Gaussian noise channel with two-sided noise power spectral density $N_o/2$ watts/Hz.

a) (**5 points**) Suppose that minimum distance decisions are used and the messages are transmitted with equal probability. Derive a union bound on the probability of symbol error in terms of E_b/N_o.

b) (**3 points**) Derive a simple upper bound on the probability of symbol error in terms of the minimum distance between signal vectors. Express your answer in terms of E_b/N_o.

c) (**2 points**) Specify a set of time domain waveforms $s_1(t)$, $s_2(t)$, $s_3(t)$ and $s_4(t)$, defined on the interval $0 \leq t \leq T$ that have the above signal vectors. *Note: The solution is not unique.*