GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
Final Examination - Fall 2013
EE 4601: Communication Systems

Aids Allowed: Course textbook
one $8\frac{1}{2} \times 11$ two-side crib sheet
Attempt all questions
All questions are of equal value

DATE: Friday December 13, 2013.

TIME: 2:50pm - 5:40pm
INSTRUCTOR: Professor G.L. Stüber

1) Random Processes: Let $X(t)$ be a Gaussian random process with zero mean and autocorrelation function

$$\phi_{XX}(\tau) = 10^{-5} B_w \text{sinc}^2(B_w \tau), \qquad B_w = 10^6 \text{ Hz}$$

The process $X(t)$ is input to a linear filter having the impulse response

$$h(t) = B\text{sinc}(2Bt), \qquad B = 10^5 \text{ Hz}$$

and the output process is $Y(t) = X(t) * h(t)$.

a) 4 marks: Sketch and fully dimension the power spectral density of $Y(t)$.

b) 2 marks: What is the probability density function of the random variable $Y(t_1)$ at any time instant t_1?

c) 4 marks: What is the probability that the output $Y(t_1)$ lies in the range $[-0.1, 0.1]$?

2) ISI Channels: Consider the transmission of a binary data sequence $\{a_n\}$, where the a_n take on values from the set $\{-1+1\}$ with equal probability, using binary antipodal signaling. The overall pulse response is $p(t) = g(t) * c(t) * h(t)$, where $g(t)$ is the transmit filter, $c(t)$ is the channel, and $h(t)$ is the receiver matched filter. The sampled pulse $p_k = p(kT)$ is such that

$$p_k = \begin{cases} 1, & k = 0 \\ 0.25, & k = 1 \\ 0.25, & k = 2 \\ 0, & \text{elsewhere} \end{cases}$$

The samples at the output of the receiver matched filter are

$$y_n = p_0 a_n + \sum_{k \neq n} a_k p_{n-k} + n_n$$

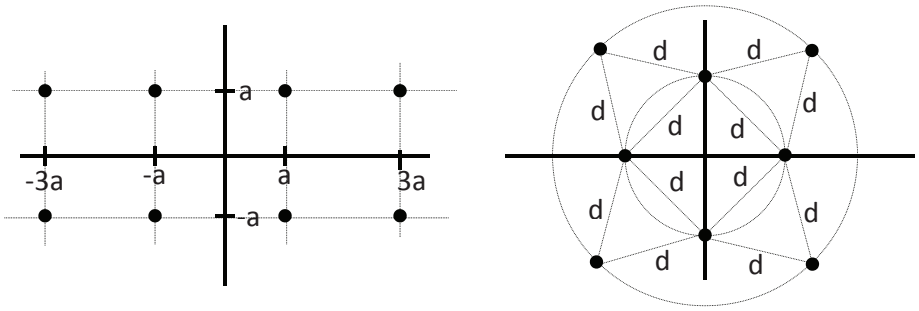
where the n_n are independent zero-mean Gaussian random variables with variance σ^2. A decision on a_n is made by only using the observation y_n, i.e., no equalization is used.

a) 4 marks: What are the possible values for the ISI term and their probabilities?

b) 4 marks: Derive an expression for the probability of bit error.

c) 2 marks: Now suppose that a zero forcing equalizer is used. Would the zero-forcing solution minimize the peak distortion after equalization?

3) Signal Constellations: Consider the two different signal constellations shown below, where each constellation has 8 points.



a) 5 marks: Find the minimum distance $d_{\min}$ of each signal constellation in terms of the average symbol energy E_{av}.

b) 3 marks: Using $d_{\min}$, derive a simple upper bound on the probability of *symbol* error for each constellation in terms of the *bit* energy-to-noise ration E_b/N_o.

c) 2 marks: At high E_b/N_o, which constellation will be better in terms of error probability performance? How much better will it be in terms of the E_b/N_o (in dB) required to give the same error probability as the other constellation?

4) Modulation: Two data streams, $\{x_n^1\}$ and $\{x_n^2\}$, $x_n^1, x_n^2 \in \{-1, +1\}$, are to be transmitted using *unbalanced* QPSK, where the data rate for $\{x_n^1\}$ is 10 kbps and that for $\{x_n^2\}$ is 1 Mbps. The unbalanced QPSK bandpass waveform is given by

$$s(t) = A_1 \sum_n x_n^1 u_{T_1}(t - nT_1) \cos(2\pi f_c t) + A_2 \sum_n x_n^2 u_{T_2}(t - nT_2) \sin(2\pi f_c t)$$

where $u_{T_i}(t) = u(t) - u(t - T_i)$, $u(t)$ is the unit step function, and n is the baud symbol index.

a) 5 marks: Relate the amplitudes of the waveforms, A_1 and A_2, such that both data streams have equal energies per bit.

b) 5 marks: With A_1 and A_2 so related, find the possible phases that the bandpass waveform can assume.

5) Error Probability: Suppose that the following signal vectors are used on an AWGN channel with two-sided noise power spectral density of $N_o/2$ watts/Hz.

$$\begin{aligned} \mathbf{s}_1 &= (a, 2a, a) \\ \mathbf{s}_2 &= (a, 0, a) \\ \mathbf{s}_3 &= (-a, 2a, a) \\ \mathbf{s}_4 &= (-a, 0, a) \end{aligned}$$

a) (**5 marks**) Find the *exact* probability of *symbol* error in terms of the average modulated *symbol* energy-to-noise ratio. *Hint: Use a graphical approach to visualize the signal vectors.*

b) (**3 marks**) Suppose that a new set of signal vectors is constructed by translating the vectors as follows:

$$\hat{\mathbf{s}}_i = \mathbf{s}_i - \mathbf{b} \ , \quad i = 1, \ldots 4$$

What vector $\mathbf{b} = (b_1, b_2, b_3)$ will minimize the average *symbol* energy in the new signal constellation $\{\hat{\mathbf{s}}_i\}_{i=1}^4$?

c) (**2 marks**) How would the symbol error probability in part b) compare with that in part a)?