GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
Final Examination - Fall 2014
EE 4601: Communication Systems

Aids Allowed: Course textbook,
one $8\frac{1}{2} \times 11$ two-side crib sheet
Attempt all questions
All questions are of equal value

DATE: Thursday December 11, 2014.
TIME: 11:30am -2:20pm
INSTRUCTOR: Professor G.L. Stüber

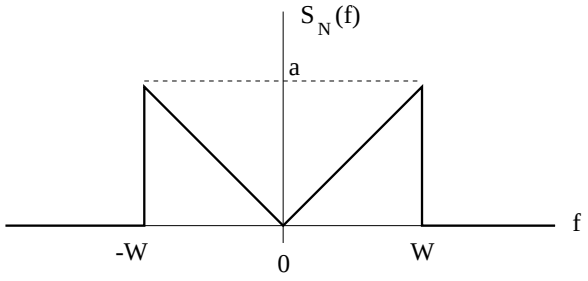
1) Random Processes: A pair of noise processes $n_1(t)$ and $n_2(t)$ are related by

$$n_2(t) = n_1(t)\cos(2\pi f_c t + \Theta) - n_1(t)\sin(2\pi f_c t + \Theta)$$

where f_c is a constant, and Θ is a random variable whose probability density function is defined by

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}\,, & 0 \le \theta \le 2\pi \\ 0\,, & \text{elsewhere} \end{cases}$$

The noise process $n_1(t)$ is wide sense stationary and its power spectral density is shown below.



a) 6 marks: Derive the autocorrelation of $n_2(t)$, $\phi_{n_2 n_2}(\tau)$, in terms of the autocorrelation of $n_1(t)$, $\phi_{n_1 n_1}(\tau)$.

b) 4 marks: Find and plot the corresponding power spectral density of $n_2(t)$.

2) Signal Space and Error Probability: Consider the set of signal waveforms $\{s_i(t)\}_{i=1}^{M}$, where the $s_i(t)$ all have duration T. The corresponding set of signal vectors $\{\mathbf{s}_i\}_{i=1}^{M}$ is comprised of all possible distinct vectors

$$\mathbf{s}_i = [s_{i1},\ s_{i2},\ \ldots,\ s_{iN}], \quad i = 1,\ 2,\ \ldots, M$$

such that

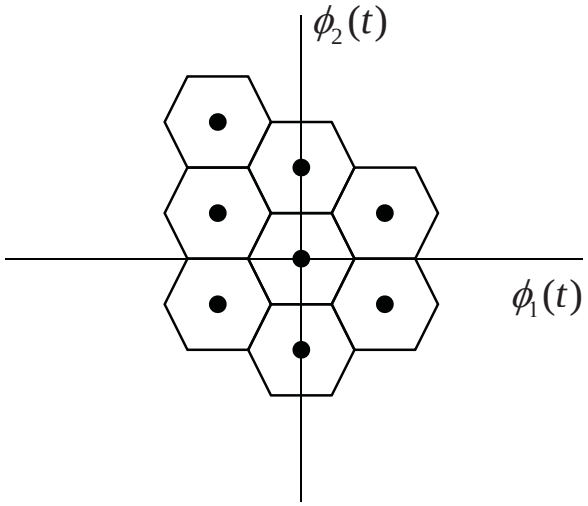
$$s_{ij} = \begin{cases} +d \\ \text{or} \\ -d \end{cases} \qquad \text{for all } i, j$$

a) 2 marks: Sketch this signal set in two ($N = 2$) and three ($N = 3$) dimensions.

b) 2 marks: How many signal vectors M are there in N dimensions?

c) 4 marks: For the case $N = 3$, derive a union bound on the probability of *symbol* error in terms of the average received *bit* energy-to-noise ratio.

3) Signal Constellations: You have been asked to design a modulation scheme to conserve bandwidth and it has been decided to use an 8-QAM signal constellation. A conventional 8-QAM constellation based on an odd-integer lattice does not use the available power efficiently. You decide on a different signal constellation. Inspired by the tile design on the floor of a local shopping mall you come up with the design shown in the figure below.



Assume that the hexagons have sides of length Δ. Determined the following:

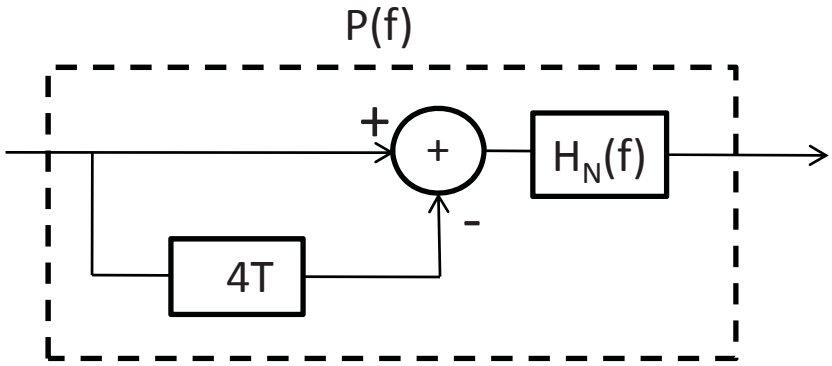
a) 2 marks: The *minimum* distance between the signals in terms of Δ.

b) 4 marks: The average transmitted energy per bit in terms of Δ.

c) 3 marks: Assuming that all signal points are equally likely, draw the decision regions of the minimum error probability receiver.

d) 3 marks: A simple upper bound on the probability of *symbol* error based on the minimum distance between signals and expressed in terms of the *bit* energy-to-noise ratio, E_b/N_o.

4) Partial Response Signals: The overall pulse response $P(f)$ of a partial response system is generated by the figure below, where $H_N(f)$ is an ideal Nyquist (low-pass) filter with a bandwidth of $W = 1/(2T)$ Hz.



a) 4 marks: What is the overall pulse response $P(f)$? Sketch the magnitude and phase response of $P(f)$, on a properly labeled graph.

b) 2 marks: What is the corresponding time domain pulse $p(t)$? Sketch it, on a properly labeled graph.

c) 4 marks: Using $P(f)$ from part a) obtain the *complex-valued* frequency responses of the corresponding transmit filter, $G(f)$, and receiver matched filter, $H(f)$.

5) Intersymbol Interference: Consider a baseband system that transmits the binary data sequence $\{a_k\}$, where $a_k \in \{-1, +1\}$ with equal probability using the rectangular pulse

$$g(t) = \sqrt{\frac{E}{T}} u_T(t) = \sqrt{\frac{E}{T}} \Big(u(t) - u(t-T) \Big)$$

where E is the energy in the pulse. The received noise corrupted waveform is applied to the matched filter $h(t)$ whose output is

$$y(t) = \sum_n a_n p(t - nT) + n(t)$$

where $p(t) = g(t) * h(t)$, $n(t) = w(t) * h(t)$, and where $w(t)$ is additive white Gaussian noise with power spectral density $N_o/2$ watts/Hz. The matched filter output $y(t)$ is sampled every T seconds to yield the sample sequence $\{y_k\}$, such that

$$y_k = y(kT + \varepsilon T) = \sum_n a_n p(kT + \varepsilon T - nT) + n_k$$

where εT is the *timing error* with $|\varepsilon| < 0.5$, and $n_k = n(kT + \varepsilon T)$.

a) 2 marks: Sketch the filter, $h(t)$, matched to the transmitted pulse $g(t) = u_T(t)$.

b) 2 marks: What is the overall pulse $p(t) = g(t) * h(t)$.

c) 4 marks: Due to the presence of a timing offset, ε, intersymbol interference (ISI) is introduced into the samples y_k. Obtain an expression for the samples y_k in terms of the sampled pulse $p_k = p(kT + \varepsilon T)$ and the noise samples n_k.

d) 2 marks: Using your results from part d) derive an expression for the probability of bit error in terms of the bit energy-to-noise ratio, E_b/N_o?