GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
Final Examination - Fall 2015
EE 4601: Communication Systems

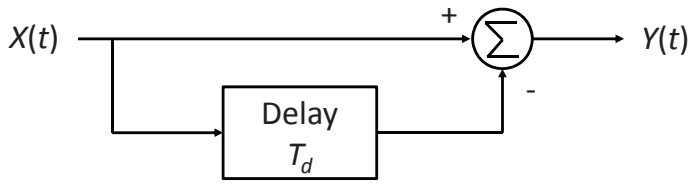
Aids Allowed: 2 8 1/2"X11" crib sheets, calculator
Attempt all questions
Questions are of equal value

DATE: Tuesday December 8, 2015.
TIME: 11:30am - 2:20pm
INSTRUCTOR: Prof. G.L. Stüber

Math tables are attached at the end of this exam
You do not need to turn them in

1) Random Processes: Suppose that a wide-sense stationary Gaussian random process $X(t)$ is input to the filter shown below. The autocorrelation function of $X(t)$ is

$$\Phi_{XX}(\tau) = \exp\{-\alpha\tau^2\}$$



a) (**4 points**) Find the power spectral density of the output random process $Y(t)$, $\Phi_{YY}(f)$.

b) (**1 points**) What frequency components are not present in $\Phi_{YY}(f)$?

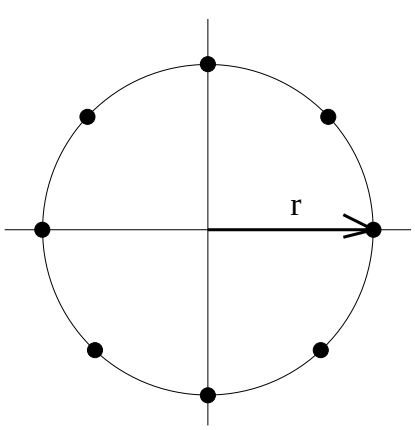
c) (**4 points**) Find the output autocorrelation function $\phi_{YY}(\tau)$.

d) (**1 points**) What is the total power in the output process $Y(t)$?

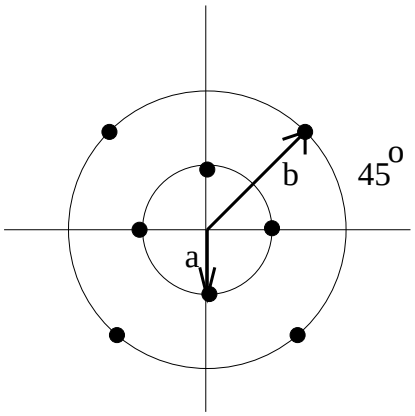
Extra sheet

Extra sheet

2) Signal Space and Error Probability: Consider the two octal signal point constellations shown in the figure below. Assume the signal points are used with equal probability.



8-PSK



8-QAM

a) (**3 points**) Each of the four inner constellation points in the 8-QAM signal constellation have 4 nearest neighbours at distance A. Determine the radii a and b of the inner and outer circles.

b) (**3 points**) The nearest neighbour signal points in the 8-PSK signal constellation are separated by a distance of A units. Determine the radius r of the circle.

c) (**2 points**) Determine the average symbol energy for the two signal constellations in terms of A. What is the relative average power advantage of 8-QAM vs. 8-PSK (in decibels) if they both have the same minimum distance A?

d) (**2 points**) For each constellation, derive a simple upper bound on the probability of symbol error by assuming that all signal constellation points are separated by the minimum distance. Express your answer in terms of E_b/N_o in each case.

Extra sheet

Extra sheet

3) Signal Constellations: Consider the following set of four signal vectors.

$$\begin{aligned}
\mathbf{s}_1 &= \sqrt{E/6}\ (-1,-1,-1,+1,+1,+1) \\
\mathbf{s}_2 &= \sqrt{E/6}\ (-1,+1,-1,-1,+1,+1) \\
\mathbf{s}_3 &= \sqrt{E/6}\ (+1,-1,-1,+1,-1,-1) \\
\mathbf{s}_4 &= \sqrt{E/6}\ (+1,-1,+1,-1,-1,-1)
\end{aligned}$$

The messages are transmitted over an additive white Gaussian noise channel with two-sided noise power spectral density $N_o/2$ watts/Hz.

a) (**4 points**) Suppose that minimum distance decisions are used and the messages are transmitted with equal probability. Derive a union bound on the probability of symbol error in terms of E_b/N_o.

b) (**4 points**) Determine the translation that will minimize the average energy in the signal constellation E_b/N_o. What is the energy savings in *decibels* by using the translated signal constellation as compared to the original signal constellation.

c) (**2 points**) Specify a set of time domain waveforms $s_1(t)$, $s_2(t)$, $s_3(t)$ and $s_4(t)$, defined on the interval $0 \leq t \leq T$ that have the above signal vectors. *Note: The solution is not unique.*

Extra sheet

Extra sheet

4) Intersymbol Interference: Consider a communication system having the overall pulse $p(t) = g(t) * c(t) * h(t)$, where $g(t)$ is the transmit filter, $c(t)$ is the channel, and $h(t)$ is the receiver filter. The transmit filter is

$$g(t) = \sqrt{\frac{E}{T}}\Big(u(t) - u(t-T)\Big)$$

and $u(t)$ is the unit step function. Suppose, that the frequency response of the channel is given by

$$C(f) = 1 + 0.5\cos(2\pi f t_o)$$

a) (**5 points**) If the filter $h(t)$ is matched to $g(t)$, find the overall pulse $p(t)$.

b) (**2 points**) If $t_o = T/2$, sketch $p(t)$ and find the coefficients of the sampled pulse $p_k = p(kT)$.

c) (**3 points**) Assuming that the $p_k, k \neq 1$ contribute to intersymbol interference (ISI), find the mean and variance of the ISI term, assuming random data symbols chosen from the set $\{-1, +1\}$ with equal probability.

Extra sheet

Extra sheet

5) Coding: The generator matrix of a systematic (7,4) Hamming code is

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

The corresponding parity check matrix is

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The **dual** to a (7,4) Hamming code can be obtained by exchanging the generator and parity check matrices, i.e., let $\mathbf{H}$ be the generator matrix and let $\mathbf{G}$ be the parity check matrix. The dual to a (7,4) Hamming code is a (7,3) Simplex code.

a) (**2 points**) List out the codewords of the (7,3) Simplex code.

b) (**2 points**) Find the weight distribution of the code.

c) (**4 points**) Suppose that the (7,3) Simplex code is used for error detection on a binary symmetric channel with crossover probability p. Find the probability of undetected error in terms of p, and evaluate for $p = 0.1$.

d) (**2 points**) The (7,3) Simplex code is capable of either detecting all patterns of s or fewer errors, or correcting all patterns of t or fewer errors. Find s and t.

Extra sheet

Extra sheet

FOURIER TRANSFORM PAIRS

Pair Number	$x(t)$	$X(f)$
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}(\tau f)$
2.	$2W\,\mathrm{sinc}(2Wt)$	$\Pi\left(\frac{f}{2W}\right)$
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}^2(\tau f)$
4.	$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$
5.	$t\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$
6.	$\exp(-\alpha\lvert t\rvert),\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$
7.	$\exp(-\alpha t^2)$	$\sqrt{\frac{\pi}{\alpha}}\exp\left(-\frac{\pi^2 f^2}{\alpha}\right)$
8.	$\delta(t)$	1
9.	1	$\delta(f)$
10.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
11.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$
12.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
13.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$
14.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$
15.	$\mathrm{sgn}(t)$	$(j\pi f)^{-1}$
16.	$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
17.	$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\lambda)}{t - \lambda}\,d\lambda$	$-j\,\mathrm{sgn}(f)X(f)$
18.	$\sum_{m=-\infty}^{\infty}\delta(t - mT_s)$	$f_s\sum_{m=-\infty}^{\infty}\delta(f - mf_s), f_s = T_s^{-1}$

Note: $\mathrm{sinc}\ u = \frac{\sin \pi u}{\pi u}$

$$\Pi(u) = \begin{cases} 1, & |u| \le 1/2 \\ 0, & \text{otherwise} \end{cases}$$

$$\Lambda(u) = \begin{cases} 1 - |u|, & |u| \le 1 \\ 0, & \text{otherwise} \end{cases}$$

FOURIER TRANSFORM THEOREMS

Name of Theorem	Signal	Transform
1. Superposition (a_1 and a_2 arbitrary constants)	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(f) + a_2X_2(f)$
2. Time delay	$x(t - t_0)$	$X(f) \exp(-j\,2\pi f t_0)$
3a. Scale change	$x(at)$	$\lvert a\rvert^{-1}X\left(\frac{f}{a}\right)$
3b. Time reversal[1]	$x(-t)$	$X(-f) = X^*(f)$
4. Duality	$X(t)$	$x(-f)$
5a. Frequency translation	$x(t) \exp(j2\pi f_0 t)$	$X(f - f_0)$
5b. Modulation	$x(t) \cos 2\pi f_0 t$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
6. Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
7. Integration	$\int_{-\infty}^{t} x(t')\,dt'$	$(j2\pi f)^{-1}X(f) + \frac{1}{2}X(0)\delta(f)$
8. Convolution	$\int_{-\infty}^{\infty} x_1(t - t')x_2(t')\,dt'$ $= \int_{-\infty}^{\infty} x_1(t')x_2(t - t')\,dt'$	$X_1(f)X_2(f)$
9. Multiplication	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty} X_1(f - f')X_2(f')\,df'$ $= \int_{-\infty}^{\infty} X_1(f')X_2(f - f')\,df'$

[1] $x(t)$ is assumed to be real in 3b.

Pythagorean relations

$$\sin^2\alpha + \cos^2\alpha = 1, \qquad 1 + \tan^2\alpha = \sec^2\alpha, \qquad 1 + \cot^2\alpha = \csc^2\alpha$$

Angle-sum and angle-difference relations

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$
$$\cot(\alpha + \beta) = \frac{\cot\beta\cot\alpha - 1}{\cot\beta + \cot\alpha}$$
$$\cot(\alpha - \beta) = \frac{\cot\beta\cot\alpha + 1}{\cot\beta - \cot\alpha}$$
$$\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$$
$$\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha$$

Double-angle relations

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}, \qquad \cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$$

Multiple-angle relations

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$
$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$
$$\sin 4\alpha = 4\sin\alpha\cos\alpha - 8\sin^3\alpha\cos\alpha$$
$$\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$$
$$\sin 5\alpha = 5\sin\alpha - 20\sin^3\alpha + 16\sin^5\alpha$$
$$\cos 5\alpha = 16\cos^5\alpha - 20\cos^3\alpha + 5\cos\alpha$$
$$\sin 6\alpha = 32\cos^5\alpha\sin\alpha - 32\cos^3\alpha\sin\alpha + 6\cos\alpha\sin\alpha$$
$$\cos 6\alpha = 32\cos^6\alpha - 48\cos^4\alpha + 18\cos^2\alpha - 1$$
$$\sin n\alpha = 2\sin(n-1)\alpha\cos\alpha - \sin(n-2)\alpha$$
$$\cos n\alpha = 2\cos(n-1)\alpha\cos\alpha - \cos(n-2)\alpha$$
$$\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$$
$$\tan 4\alpha = \frac{4\tan\alpha - 4\tan^3\alpha}{1 - 6\tan^2\alpha + \tan^4\alpha}$$
$$\tan n\alpha = \frac{\tan(n-1)\alpha + \tan\alpha}{1 - \tan(n-1)\alpha\tan\alpha}$$

Formulas for Use in Trigonometry

Function-product relations

$$\sin\alpha\sin\beta = \tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha + \beta) + \tfrac{1}{2}\sin(\alpha - \beta)$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\sin(\alpha + \beta) - \tfrac{1}{2}\sin(\alpha - \beta)$$

Function-sum and function-difference relations

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$
$$\sin\alpha - \sin\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$
$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$
$$\cos\alpha - \cos\beta = -2\sin\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$
$$\tan\alpha + \tan\beta = \frac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta}, \qquad \tan\alpha - \tan\beta = \frac{\sin(\alpha - \beta)}{\cos\alpha\cos\beta}$$
$$\cot\alpha + \cot\beta = \frac{\sin(\alpha + \beta)}{\sin\alpha\sin\beta}, \qquad \cot\alpha - \cot\beta = \frac{\sin(\beta - \alpha)}{\sin\alpha\sin\beta}$$
$$\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{\tan\tfrac{1}{2}(\alpha + \beta)}{\tan\tfrac{1}{2}(\alpha - \beta)}, \qquad \frac{\sin\alpha + \sin\beta}{\cos\alpha - \cos\beta} = \cot\tfrac{1}{2}(\beta - \alpha)$$
$$\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \tan\tfrac{1}{2}(\alpha + \beta), \qquad \frac{\sin\alpha - \sin\beta}{\cos\alpha + \cos\beta} = \tan\tfrac{1}{2}(\alpha - \beta)$$

Half-angle relations

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}, \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}} = \frac{1 - \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 + \cos\alpha}$$
$$\cot\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{1 - \cos\alpha}} = \frac{1 + \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 - \cos\alpha}$$

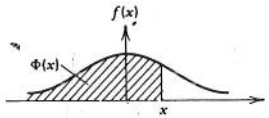
Power relations

$$\sin^2\alpha = \tfrac{1}{2}(1 - \cos 2\alpha), \qquad \sin^3\alpha = \tfrac{1}{4}(3\sin\alpha - \sin 3\alpha)$$
$$\sin^4\alpha = \tfrac{1}{8}(3 - 4\cos 2\alpha + \cos 4\alpha)$$
$$\cos^2\alpha = \tfrac{1}{2}(1 + \cos 2\alpha), \qquad \cos^3\alpha = \tfrac{1}{4}(3\cos\alpha + \cos 3\alpha)$$
$$\cos^4\alpha = \tfrac{1}{8}(3 + 4\cos 2\alpha + \cos 4\alpha)$$
$$\tan^2\alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}, \qquad \cot^2\alpha = \frac{1 + \cos 2\alpha}{1 - \cos 2\alpha}$$

Exponential relations (α in radians), Euler's equation

$$e^{i\alpha} = \cos\alpha + i\sin\alpha, \qquad i = \sqrt{-1}$$
$$\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}, \qquad \cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$
$$\tan\alpha = -i\left(\frac{e^{i\alpha} - e^{-i\alpha}}{e^{i\alpha} + e^{-i\alpha}}\right) = -i\left(\frac{e^{2i\alpha} - 1}{e^{2i\alpha} + 1}\right)$$

APPENDIX D



*Tabulation of the Standard Normal Distribution**

x	Φ(x)	f(x)	x	Φ(x)	f(x)	x	Φ(x)	f(x)
.00	.50000	.39894	.50	.69146	.35207	1.00	.84134	.24197
.01	.50399	.39892	.51	.69497	.35029	1.01	.84375	.23955
.02	.50798	.39886	.52	.69847	.34849	1.02	.84614	.23713
.03	.51197	.39876	.53	.70194	.34667	1.03	.84850	.23471
.04	.51595	.39862	.54	.70540	.34482	1.04	.85083	.23230
.05	.51994	.39844	.55	.70884	.34294	1.05	.85314	.22988
.06	.52392	.39822	.56	.71226	.34105	1.06	.85543	.22747
.07	.52790	.39797	.57	.71566	.33912	1.07	.85769	.22506
.08	.53188	.39767	.58	.71904	.33718	1.08	.85993	.22265
.09	.53586	.39733	.59	.72240	.33521	1.09	.86214	.22025
.10	.53983	.39695	.60	.72575	.33322	1.10	.86433	.21785
.11	.54380	.39654	.61	.72907	.33121	1.11	.86650	.21546
.12	.54776	.39608	.62	.73237	.32918	1.12	.86864	.21307
.13	.55172	.39559	.63	.73565	.32713	1.13	.87076	.21069
.14	.55567	.39505	.64	.73891	.32506	1.14	.87286	.20831
.15	.55982	.39448	.65	.74215	.32297	1.15	.87493	.20594
.16	.56356	.39387	.66	.74537	.32086	1.16	.87698	.20357
.17	.56750	.39322	.67	.74857	.31874	1.17	.87900	.20121
.18	.57142	.39253	.68	.75175	.31659	1.18	.88100	.19886
.19	.57535	.39181	.69	.75490	.31443	1.19	.88298	.19652
.20	.57926	.39104	.70	.75804	.31225	1.20	.88493	.19419
.21	.58317	.39024	.71	.76115	.31006	1.21	.88686	.19186
.22	.58706	.38940	.72	.76424	.30785	1.22	.88877	.18954
.23	.59095	.38853	.73	.76730	.30563	1.23	.89065	.18724
.24	.59484	.38762	.74	.77035	.30339	1.24	.89251	.18494
.25	.59871	.38667	.75	.77337	.30114	1.25	.89435	.18265
.26	.60257	.38568	.76	.77637	.29887	1.26	.89617	.18037
.27	.60643	.38466	.77	.77935	.29659	1.27	.89796	.17810
.28	.61026	.38361	.78	.78230	.29431	1.28	.89973	.17585
.29	.61409	.38251	.79	.78524	.29200	1.29	.90147	.17360
.30	.61791	.38139	.80	.78814	.28969	1.30	.90320	.17137
.31	.62172	.38023	.81	.79103	.28737	1.31	.90490	.16915
.32	.62552	.37903	.82	.79389	.28504	1.32	.90658	.16694
.33	.62930	.37780	.83	.79673	.28269	1.33	.90824	.16474
.34	.63307	.37654	.84	.79955	.28034	1.34	.90988	.16256
.35	.63683	.37524	.85	.80234	.27798	1.35	.91149	.16038
.36	.64058	.37391	.86	.80511	.27562	1.36	.91308	.15822
.37	.64431	.37255	.87	.80785	.27324	1.37	.91466	.15608
.38	.64803	.37115	.88	.81057	.27086	1.38	.91621	.15395
.39	.65173	.36973	.89	.81327	.26848	1.39	.91774	.15183
.40	.65542	.36827	.90	.81594	.26609	1.40	.91924	.14973
.41	.65910	.36678	.91	.81859	.26369	1.41	.92073	.14764
.42	.66276	.36526	.92	.82121	.26129	1.42	.92220	.14556
.43	.66640	.36371	.93	.82381	.25888	1.43	.92364	.14350
.44	.67003	.36213	.94	.82639	.25647	1.44	.92507	.14146
.45	.67365	.36053	.95	.82894	.25406	1.45	.92647	.13943
.46	.67724	.35889	.96	.83147	.25164	1.46	.92786	.13742
.47	.68082	.35723	.97	.83398	.24923	1.47	.92922	.13542
.48	.68439	.35553	.98	.83646	.24681	1.48	.93056	.13344
.49	.68793	.35381	.99	.83891	.24439	1.49	.93189	.13147
.50	.69146	.35207	1.00	.84134	.24197	1.50	.93319	.12952

* Abridged from *Biometrika Tables for Statisticians*, vol. 1 (2nd edition), edited by E. S. Pearson and H. O. Hartley, Cambridge University Press, London, 1958, table 1, with permission of the Biometrika Trustees.

x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$
1.50	.93319	.12952	2.00	.97725	.05399	2.50	.99379	.01753
1.51	.93448	.12758	2.01	.97778	.05292	2.51	.99396	.01709
1.52	.93574	.12566	2.02	.97831	.05186	2.52	.99413	.01667
1.53	.93699	.12376	2.03	.97882	.05082	2.53	.99430	.01625
1.54	.93822	.12188	2.04	.97932	.04980	2.54	.99446	.01585
1.55	.93943	.12001	2.05	.97982	.04879	2.55	.99461	.01545
1.56	.94062	.11816	2.06	.98030	.04780	2.56	.99477	.01506
1.57	.94179	.11632	2.07	.98077	.04682	2.57	.99492	.01468
1.58	.94295	.11450	2.08	.98124	.04586	2.58	.99506	.01431
1.59	.94408	.11270	2.09	.98169	.04491	2.59	.99520	.01394
1.60	.94520	.11092	2.10	.98214	.04398	2.60	.99534	.01358
1.61	.94630	.10915	2.11	.98257	.04307	2.61	.99547	.01323
1.62	.94738	.10741	2.12	.98300	.04217	2.62	.99560	.01289
1.63	.94845	.10567	2.13	.98341	.04128	2.63	.99573	.01256
1.64	.94950	.10396	2.14	.98382	.04041	2.64	.99585	.01223
1.65	.95053	.10226	2.15	.98422	.03955	2.65	.99598	.01191
1.66	.95154	.10059	2.16	.98461	.03871	2.66	.99609	.01160
1.67	.95254	.09893	2.17	.98500	.03788	2.67	.99621	.01130
1.68	.95352	.09728	2.18	.98537	.03706	2.68	.99632	.01100
1.69	.95449	.09566	2.19	.98574	.03626	2.69	.99643	.01071
1.70	.95543	.09405	2.20	.98610	.03547	2.70	.99653	.01042
1.71	.95637	.09246	2.21	.98645	.03470	2.71	.99664	.01014
1.72	.95728	.09089	2.22	.98679	.03394	2.72	.99674	.00987
1.73	.95818	.08933	2.23	.98713	.03319	2.73	.99683	.00961
1.74	.95907	.08780	2.24	.98745	.03246	2.74	.99693	.00935
1.75	.95994	.08628	2.25	.98778	.03174	2.75	.99702	.00909
1.76	.96080	.08478	2.26	.98809	.03103	2.76	.99711	.00885
1.77	.96164	.08329	2.27	.98840	.03034	2.77	.99720	.00861
1.78	.96246	.08183	2.28	.98870	.02965	2.78	.99728	.00837
1.79	.96327	.08038	2.29	.98899	.02898	2.79	.99736	.00814
1.80	.96407	.07895	2.30	.98928	.02833	2.80	.99744	.00792
1.81	.96485	.07754	2.31	.98956	.02768	2.81	.99752	.00770
1.82	.96562	.07614	2.32	.98983	.02705	2.82	.99760	.00748
1.83	.96638	.07477	2.33	.99010	.02643	2.83	.99767	.00727
1.84	.96712	.07341	2.34	.99036	.02582	2.84	.99774	.00707
1.85	.96784	.07206	2.35	.99061	.02522	2.85	.99781	.00687
1.86	.96856	.07074	2.36	.99086	.02463	2.86	.99788	.00668
1.87	.96926	.06943	2.37	.99111	.02406	2.87	.99795	.00649
1.88	.96995	.06814	2.38	.99134	.02349	2.88	.99801	.00631
1.89	.97062	.06687	2.39	.99158	.02294	2.89	.99807	.00613
1.90	.97128	.06562	2.40	.99180	.02239	2.90	.99813	.00595
1.91	.97193	.06438	2.41	.99202	.02186	2.91	.99819	.00578
1.92	.97257	.06316	2.42	.99224	.02134	2.92	.99825	.00562
1.93	.97320	.06195	2.43	.99245	.02083	2.93	.99831	.00545
1.94	.97381	.06077	2.44	.99266	.02033	2.94	.99836	.00530
1.95	.97441	.05959	2.45	.99286	.01984	2.95	.99841	.00514
1.96	.97500	.05844	2.46	.99305	.01936	2.96	.99846	.00499
1.97	.97558	.05730	2.47	.99324	.01889	2.97	.99851	.00485
1.98	.97615	.05618	2.48	.99343	.01842	2.98	.99856	.00471
1.99	.97670	.05508	2.49	.99361	.01797	2.99	.99861	.00457
2.00	.97725	.05399	2.50	.99379	.01753	3.00	.99865	.00443

x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$
3.00	.99865	.00443	3.50	.99977	.00087	4.00	.99997	.00013
3.01	.99869	.00430	3.51	.99978	.00084	4.01	.99997	.00013
3.02	.99874	.00417	3.52	.99978	.00081	4.02	.99997	.00012
3.03	.99878	.00405	3.53	.99979	.00079	4.03	.99997	.00012
3.04	.99882	.00393	3.54	.99980	.00076	4.04	.99997	.00011
3.05	.99886	.00381	3.55	.99981	.00073	4.05	.99997	.00011
3.06	.99889	.00370	3.56	.99981	.00071	4.06	.99998	.00011
3.07	.99893	.00358	3.57	.99982	.00068	4.07	.99998	.00010
3.08	.99897	.00348	3.58	.99983	.00066	4.08	.99998	.00010
3.09	.99900	.00337	3.59	.99983	.00063	4.09	.99998	.00009
3.10	.99903	.00327	3.60	.99984	.00061	4.10	.99998	.00009
3.11	.99906	.00317	3.61	.99985	.00059	4.11	.99998	.00009
3.12	.99910	.00307	3.62	.99985	.00057	4.12	.99998	.00008
3.13	.99913	.00298	3.63	.99986	.00055	4.13	.99998	.00008
3.14	.99916	.00288	3.64	.99986	.00053	4.14	.99998	.00008
3.15	.99918	.00279	3.65	.99987	.00051	4.15	.99998	.00007
3.16	.99921	.00271	3.66	.99987	.00049	4.16	.99998	.00007
3.17	.99924	.00262	3.67	.99988	.00047	4.17	.99998	.00007
3.18	.99926	.00254	3.68	.99988	.00046	4.18	.99999	.00006
3.19	.99929	.00246	3.69	.99989	.00044	4.19	.99999	.00006
3.20	.99931	.00238	3.70	.99989	.00042	4.20	.99999	.00006
3.21	.99934	.00231	3.71	.99990	.00041	4.21	.99999	.00006
3.22	.99936	.00224	3.72	.99990	.00039	4.22	.99999	.00005
3.23	.99938	.00216	3.73	.99990	.00038	4.23	.99999	.00005
3.24	.99940	.00210	3.74	.99991	.00037	4.24	.99999	.00005
3.25	.99942	.00203	3.75	.99991	.00035	4.25	.99999	.00005
3.26	.99944	.00196	3.76	.99992	.00034	4.26	.99999	.00005
3.27	.99946	.00190	3.77	.99992	.00033	4.27	.99999	.00004
3.28	.99948	.00184	3.78	.99992	.00031	4.28	.99999	.00004
3.29	.99950	.00178	3.79	.99992	.00030	4.29	.99999	.00004
3.30	.99952	.00172	3.80	.99993	.00029	4.30	.99999	.00004
3.31	.99953	.00167	3.81	.99993	.00028	4.31	.99999	.00004
3.32	.99955	.00161	3.82	.99993	.00027	4.32	.99999	.00004
3.33	.99957	.00156	3.83	.99994	.00026	4.33	.99999	.00003
3.34	.99958	.00151	3.84	.99994	.00025	4.34	.99999	.00003
3.35	.99960	.00146	3.85	.99994	.00024	4.35	.99999	.00003
3.36	.99961	.00141	3.86	.99994	.00023	4.36	.99999	.00003
3.37	.99962	.00136	3.87	.99995	.00022	4.37	.99999	.00003
3.38	.99964	.00132	3.88	.99995	.00021	4.38	.99999	.00003
3.39	.99965	.00127	3.89	.99995	.00021	4.39	.99999	.00003
3.40	.99966	.00123	3.90	.99995	.00020	4.40	.99999	.00002
3.41	.99968	.00119	3.91	.99995	.00019	4.41	.99999	.00002
3.42	.99969	.00115	3.92	.99996	.00018	4.42	1.00000	.00002
3.43	.99970	.00111	3.93	.99996	.00018	4.43	1.00000	.00002
3.44	.99971	.00107	3.94	.99996	.00017	4.44	1.00000	.00002
3.45	.99972	.00104	3.95	.99996	.00016	4.45	1.00000	.00002
3.46	.99973	.00100	3.96	.99996	.00016	4.46	1.00000	.00002
3.47	.99974	.00097	3.97	.99996	.00015	4.47	1.00000	.00002
3.48	.99975	.00094	3.98	.99997	.00014	4.48	1.00000	.00002
3.49	.99976	.00090	3.99	.99997	.00014	4.49	1.00000	.00002
3.50	.99977	.00087	4.00	.99997	.00013	4.50	1.00000	.00002