Georgia Institute of Technology
School of Electrical and Computer Engineering

# ECE6604 Personal & Mobile Communications

Final Exam

April 28, 2004
8:00am - 10:50am

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

1) A brief measurement campaign indicates that the propagation path loss at 420 MHz in a midsize North American city can be modelled with $\beta = 2.8$ and a fixed loss of 25 dB; that is,

$$L_p = 25dB + 10\log_{10}\left(d^{2.8}\right)$$

The receiver sensitivity is -95 dBm and the transmit and receiver antenna gains are 0 dB.

**a) 4 marks:** Ignoring shadowing, what transmitter power is required to service a circular area of radius 10 km?

**b) 3 marks:** Suppose that the measurements were optimistic and $\beta = 3.1$ is more appropriate. What is the corresponding increase in transmit power that would be required?

**b) 3 marks:** Now suppose that log-normal shadowing is present with a standard deviation of 6 dB. Assume a single isolated cell (no handoff gain available) and a requirement of 10% outage on the cell boundary. What is the required increase in transmit power over the value obtained in part a) if a service area of radius 10 km is to be maintained?

2) Consider a system where the area mean signal strength is described by the equation

$$\Omega_{p\ (\mathrm{dBm})}(d) = \mu_{\Omega_{p\ (\mathrm{dBm})}}(d_o) - 10\beta \log_{10}(d/d_o) + \epsilon_{(\mathrm{dB})} \quad (\mathrm{dBm})$$

where $d_o = 1$ m, $\mu_{\Omega_{p\ (\mathrm{dBm})}}(d_o) = 0$ dBm, $\beta = 3.5$, and $\epsilon_{(\mathrm{dB})}$ is a zero-mean Gaussian random variable with standard deviation $\sigma_\Omega = 8$ dB.

**a) 6 marks:** Suppose that a mobile station is 1 km from its serving base-station and at distances 3 km and 4 km from two co-channel base-stations. Using the Fenton Wilkinson method derive the probability density function of the received interference power $I$ in decibel units.

**a) 4 marks:** What is the probability density function of the received carrier-to-interference ratio $C/I$ in decibel units.

3) Two data streams, $d_1(t)$ and $d_2(t)$, are to be transmitted using *unbalanced* QPSK, where the data rate for $d_1(t)$ is 10 kbps and that for $d_2(t)$ is 1 Mbps.

**a) 6 marks:** Relate the amplitudes of the waveforms, $A_1$ and $A_2$, such that both bit sequences have equal energies per bit.

**b) 4 marks:** With $A_1$ and $A_2$ so related, find the possible phase shifts for the carrier, where $d_1(t)$ and $d_2(t)$ take on all possible combinations of $+1$ and $-1$.

4) Consider digital transmission via BPSK modulation with second-order receiver diversity. The complex channel gain in each branch takes on a value of 1.0 (real valued), with probability 0.9, and a value of 0.05 (real valued), with probability 0.1, independently on each symbol interval. Each receiver branch is affected by AWGN with two-sided noise power spectral density $N_o/2$ watts/Hz. The two diversity branches have independent channel gains and noise components.

   **a) 10 marks:** Derive the bit error rate performance with maximal ratio combining.

5) An OFDM signal with a large number of sub-carriers $N$ has a complex envelope that can be approximated as a zero-mean complex Gaussian random process. Assume an "ideal" OFDM signal spectrum, where the power spectrum is

$$S_{\tilde{s}\tilde{s}}(f) = \begin{cases} S_0 & |f| \leq 1/2T_s \\ 0 & \text{elsewhere} \end{cases}$$

where $T = NT_s$.

**a) 4 marks:** Using the above Gaussian approximation, what is the distribution of the complex envelope $\tilde{s}(t)$ at any time $t$.

**b) 3 marks:** Suppose that a power amplifier will clip the OFDM waveform if the complex envelope $\tilde{s}(t)$ will exceeds the level $\Theta R_{\text{rms}}$, where $R_{\text{rms}}$ is the rms envelope level $\sqrt{E[|\tilde{s}(t)|^2]}$. What is the probability that the OFDM waveform will be clipped at any time $t$?

**b) 3 marks:** How many times per second will the envelope be clipped?