Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Final Exam

May 2, 2005
8:00am - 10:50am

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

1) Suppose that we transmit an unmodulated 900 MHz carrier from a fixed base station over a hypothetical radio propagation channel. The received complex envelope at a mobile station is $r(t) = r_I(t) + jr_Q(t)$, where

$$
\begin{aligned}
S_{r_I r_I}(f) &= \begin{cases} \text{rect}\left(\frac{f}{100}\right), & |f| \leq 50\ Hz \\ 0, & \text{elsewhere} \end{cases} \\
S_{r_I r_Q}(f) &= 0
\end{aligned}
$$

a) **2 marks:** What is the speed of the mobile station?

b) **4 marks:** If antenna diversity is deployed at the mobile station, what are the possible spatial separations between the antenna elements such that the corresponding faded envelopes will be perfectly uncorrelated?

b) **4 marks:** What is the level crossing rate?

2) Consider a wide-sense stationary channel with uncorrelated scattering (WS-SUS). The scattering function is given by

$$\psi_S(\tau,\nu) = Ce^{-|\nu|/40-40\tau} \quad , \quad -\infty < \nu < \infty, \quad 0 < \tau < \infty$$

a) **2 marks:** What is the average delay of the channel?

a) **2 marks:** What is the 3 dB Doppler spread of the channel?

a) **4 marks:** Find an expression for the spaced-time, spaced-frequency correlation function $\phi_T(\Delta t, \Delta f)$.

a) **2 marks:** If a digital communication system were to operate over this channel with a baud rate of 1000 symbols/s, would you consider the channel to be frequency selective or frequency non-selective?

4) Consider Alamouti's transmit diversity scheme. We have seen in class and in the textbook how to construct the *combiner* for the case of 2×1 and 2×2.

   a) **10 marks:** Extend Alamouti's results by deriving an expression for the combiner for the case of $2 \times L$ (2 transmit and L receive) antennas.

4) Consider the reception of a signal in the presence of a single co-channel interferer and neglect the effect of AWGN. The received signal power, C, and interference power, I, due to Rayleigh fading have the exponential distributions

$$\begin{aligned} p_C(x) &= \frac{1}{\bar{C}} e^{-x/\bar{C}} \\ p_I(y) &= \frac{1}{\bar{I}} e^{-x/\bar{I}} \end{aligned}$$

where $\bar{C}$ and $\bar{I}$ are the average received signal power and interference power, respectively.

a) 6 marks: Assuming that C and I are independent random variables, find the probability density function for the carrier-to-interference ratio

$$\lambda = \frac{C}{I} \ .$$

Hint: If X and Y are independent random variables, then the probability density function of $U = X/Y$ is

$$p_U(u) = \int p_{XY}(v, v/u)|v/u^2|dv \ .$$

b) 4 marks: Now suppose that the system uses 2-branch *selection* diversity. The selective combiner always picks the branch with the largest carrier-to-interference ratio. The branches are independent and balanced (i.e., the distribution $p_U(u)$ is the same for each branch. What is the probability density function of λ at the output of the selective combiner?

5) Consider the length-7 Barker sequence

$$\mathbf{a} = (+1, +1, +1, -1, -1, +1, -1)$$

a) 3 marks: Find and sketch the full period autocorrelation function of the sequence.

b) 3 marks: Find and sketch the aperiodic autocorrelation function of the sequence.

c) 4 marks: Sketch the DTFT of the aperiodic autocorrelation function, $\Phi_{a,a}(f)$, and find the frequencies where it attains its maximum and minimum values.