Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Final Exam

Spring 2006

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

1) Consider two complex faded envelopes $g_i(t) = g_i^I(t) + jg_i^Q(t)$, $i = 1, 2$, statistical time correlation

$$
\begin{aligned}
\boldsymbol{\mu}_{\mathbf{g}} &= \mathrm{E}[\mathbf{g}(t)] = \mathbf{0} \\
\Phi_{\mathbf{g}}(\tau) &= \frac{1}{2}\mathrm{E}[\mathbf{g}^H(t)\mathbf{g}(t+\tau)] = J_o(2\pi f_m \tau)\boldsymbol{\Sigma}^2
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{g}(t) &= (g_1(t), g_2(t)0 \\
\boldsymbol{\Sigma}^2 &= \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}
\end{aligned}
$$

We generate a third faded envelope $g_3(t)$ by combining $g_1(t)$ and $g_2(t)$:

$$
g_3(t) = \alpha g_1(t) + (1-\alpha)g_2(t)\ , \qquad\qquad 0 \leq \alpha \leq 1
$$

a) 6 marks: Compute the statistical time correlation functions

$$
\begin{aligned}
\phi_{g_1 g_3}(\tau) &= \frac{1}{2}\mathrm{E}[g_1^*(t)g_3(t+\tau)] \\
\phi_{g_2 g_3}(\tau) &= \frac{1}{2}\mathrm{E}[g_2^*(t)g_3(t+\tau)] \\
\phi_{g_3 g_3}(\tau) &= \frac{1}{2}\mathrm{E}[g_3^*(t)g_3(t+\tau)]
\end{aligned}
$$

b) 4 marks: What is the envelope (amplitude and phase) distribution of $g_3(t)$?

2) You are asked to design a highway cell reuse pattern as shown in Figure 1. Each cell has length $2R$.

a) 4 marks: A base station with an omnidirectional antenna is placed at the center of each cell. Shadowing is ignored under assumed line-of-sight conditions. Derive an expression for C/I in terms of the linear reuse cluster size N. Afterwards, determine the minimum reuse factor needed to yield a worst case carrier-to-interference ratio (C/I) of 9 dB or more. Consider only the nearest neighbouring co-channel interferers and assume a propagation path loss exponent $\beta = 4$.

b) 3 marks: Now suppose that directional antennas are used to divide each cell into two semicircular sectors with boundaries perpendicular to the highway. Repeat part a).

c) 3 marks: Consider again the sectored cell arrangement in part b). However, log-normal shadowing (due to heavy vehicle traffic) is now present with shadow standard deviation of σ_Ω dB. Derive an expression for the C/I outage on a cell boundary, again considering only the nearest neighbouring co-channel interferers.

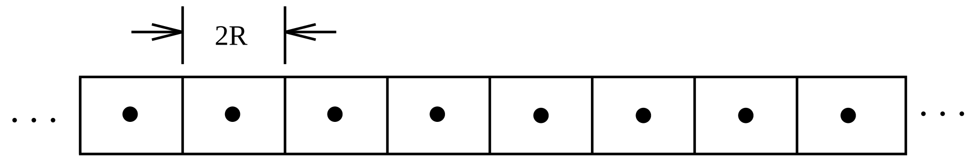

Figure 1: Figure 1: Highway cell reuse pattern.

3) Consider an OFDM time-domain sequence (without guard interval)

$$
\begin{aligned}
X_k &= \sum_{n=0}^{N-1} x_n \exp\left\{\frac{j2\pi nk}{N}\right\} \\
&= \sum_{n=0}^{N-1} x_n \cos\left(\frac{2\pi nk}{N}\right) + j\sum_{n=0}^{N-1} x_n \sin\left(\frac{2\pi nk}{N}\right), \quad k = 0,\ 1,\ \ldots,\ N-1
\end{aligned}
$$

where the x_n are symbols are i.i.d. symbols chosen from the binary alphabet $\{-1, +1\}$.

a) 4 marks: Invoke the central limit theorem for large N and treat the $X_k = X_k^I + jX_k^Q$ as independent complex Gaussian random variables. What are the mean, variance and cross-correlation of the quadrature components of X_k?

b) 3 marks: We have established that the X_k are complex Gaussian random variables, with known statistics. What is the probability density function of the peak power

$$\mathrm{P}_{\max} = |X_{\max}|^2 = \max\nolimits_{1\leq k\leq n}|X_k|^2?$$

c) 3 marks: What is the probability density function of the peak to average power ratio

$$\mathrm{PAPR} = \frac{\mathrm{P}_{\max}}{\mathrm{P}_{\mathrm{av}}}$$

in terms of the block size N?

4) Consider binary, orthogonal signaling using non-coherent FSK modulation and demodulation. The bit error rate for non-coherent FSK on an AWGN channel is

$$P_b = \frac{1}{2} e^{-\gamma_b/2}$$

where $\gamma_b = \alpha^2 E_b/N_o$ is the received bit energy-to-noise ratio. Derive the bit-error-rate for

a) 5 marks: a flat Rayleigh fading channel

b) 5 marks: a flat Ricean fading channel

5) Consider a coherent MSK system that operates over a quasi-static flat Rayleigh fading channel in the presence of a single flat Rayleigh faded co-channel interferer. The instantaneous received carrier-to-interference ratio is defined as $\lambda = s_0/s_1$, where s_0 and s_1 are independent exponentially distributed random variables with density

$$p_{\alpha^2}(x) = \frac{1}{\Omega_p} \exp\left\{-\frac{x}{\Omega_p}\right\} \ .$$

The average signal-to-interference ratio is defined as

$$\bar{\lambda} \triangleq \frac{\bar{s}_0}{\bar{s}_1}$$

a) 6 marks: Derive an expression for the probability density function of λ. You might want to consult a probability textbook reference looking for functions of random variables. What is the mean value of λ?

b) 4 marks: To improve the bit error probability performance, L-branch antenna diversity is employed at the receiver. Assume that the λ_i experienced on each of the diversity branches are independent. If selective diversity combining is used, what is the probability density function of the signal-to-interference ratio at the output of the selective combiner?