Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Final Exam

Spring 2007

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

1) A receiver is affected by 3 log-normally shadowed co-channel signals having the power sum

$$I = \sum_{k=1}^{3} I_k$$

where

$$\begin{aligned} I_{1(\mathrm{dB})} &\sim N(-10\mathrm{dBm}, 8\mathrm{dB}) \\ I_{2(\mathrm{dB})} &\sim N(-10\mathrm{dBm}, 8\mathrm{dB}) \\ I_{3(\mathrm{dB})} &\sim N(-10\mathrm{dBm}, 8\mathrm{dB}) \end{aligned}$$

where $N(\mu, \sigma)$ refers a Gaussian random variable with mean μ and variance σ^2. The sum I is to be approximated as another log-normal random variable, Z, by using the Fenton-Wilkinson method.

a) 6 marks: Find the mean and variance of $Z_{(\mathrm{dB})}$.

b) 4 marks: Suppose that the received carrier power $C_{(\mathrm{dB})}$ has the distribution

$$C_{(\mathrm{dB})} \sim N(0\mathrm{dBm}, 8\mathrm{dB})$$

Using your result from part a), what is the distribution of the carrier-to-interference ratio $(C/I)_{(\mathrm{dB})}$?

2a) 5 marks: A mobile operating at a carrier frequency of 900 MHz is experiencing Rayleigh with 2-D isotropic scattering. Calculate the number of fading events per second having the instantaneous received power 10 and 20 dB lower than the average received power for vehicle speeds of 10 and 100 km/h. Calculate the average duration of these fading events.

2b) 5 marks: Consider two propagation paths that are identical except that each experiences independent log-normal shadow fading. Write down an expression for the probability density of the shadow fading *in decibel units* at the output of the selector function.

3) Consider a system that uses binary differential phase shift keyed modulation (DPSK). The error probability on an AWGN channel is

$$P_b(\gamma_b) = \frac{1}{2} e^{-\gamma_b}$$

where $\gamma_b = \alpha^2 E_b / N_o$ is the received bit-energy-to-noise ratio.

a) 6 marks: Derive an expression for the probability of bit error for DPSK on a slow flat rayleigh fading channel. Simplify to closed form.

b) 4 marks: Repeat part a) for a slow flat Rician fading channel. Simplify as much as possible.

4) An OFDM signal with a large number of sub-carriers N has a complex envelope that can be approximated as a zero-mean complex Gaussian random process. Assume an "ideal" OFDM signal spectrum, where the power spectrum is

$$S_{\tilde{s}\tilde{s}}(f) = \begin{cases} S_0 & |f| \leq 1/2T_s \\ 0 & \text{elsewhere} \end{cases}$$

where $T = NT_s$.

a) 4 marks: Using the above Gaussian approximation, what is the distribution of the magnitude of the complex envelope $\tilde{s}(t)$ at any time t.

b) 3 marks: Suppose that a power amplifier will clip the OFDM waveform if the complex envelope $\tilde{s}(t)$ will exceeds the level ΘR_{rms}, where R_{rms} is the rms envelope level $\sqrt{E[|\tilde{s}(t)|^2]}$. What is the probability that the OFDM waveform will be clipped at any time t?

b) 3 marks: How many times per second will the envelope be clipped?

5) Consider the length-7 Barker sequence

$$\mathbf{a} = (+1, +1, +1, -1, -1, +1, -1)$$

and its mirror image sequence

$$\mathbf{a} = (-1, +1, -1, -1, +1, +1, +1)$$

a) 2 marks: Find and sketch the full period autocorrelation functions of the sequences.

b) 2 marks: Find and sketch the aperiodic autocorrelation function of the sequences.

c) 2 marks: Find and sketch the full period cross-correlation function of the two sequences.

d) 4 marks: Sketch the DTFT of the aperiodic autocorrelation function, $\Phi_{a,a}(f)$, and find the frequencies where it attains its maximum and minimum values.