Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Final Exam

Spring 2010

Tuesday May 6, 11:30am - 2:20pm

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

1a) 5 marks: The LCR at the normalized threshold ρ for a 2-D isotropic scattering channel can be expressed as

$$L_R = \sqrt{2\pi} f_m \rho e^{-\rho^2} \quad ,$$

where

$$\rho = \frac{R}{\sqrt{\Omega_p}} = \frac{R}{R_{\text{rms}}}$$

and $R_{\text{rms}} \triangleq \sqrt{\mathrm{E}[\alpha^2]}$ is the *rms* envelope level.

- **i)** Find the normalized threshold level ρ_o at which the LCR reaches its maximum value.
- **ii)** Explain why the LCR at ρ decreases as ρ deviates from ρ_o.

1b) 5 marks: Consider a cellular system with a carrier frequency of 2 GHz. Suppose that the user is in a vehicle travelling at 60 km/h. Assuming that the channel is characterized by 2D isotropic scattering, find

- **i)** the LCR at the normalized level $\rho = -3$ dB.
- **ii)** the AFD at the normalized level $\rho = -3$ dB.

2) The power delay profile for a WSSUS channel is given by

$$\phi_{gg}(\tau) = \begin{cases} 0.5[1+\cos(2\pi\tau/T)]\ , & 0 \le \tau \le T/2 \\ 0\ , & \text{otherwise} \end{cases}$$

a) 3 marks: Find the channel frequency correlation function.

b) 4 marks: Calculate the mean delay and rms delay spread.

c) 3 marks: If $T = 0.1$ ms, determine whether the channel exhibits frequency-selective fading to the GSM system.

3) Cellular CDMA systems use soft handoff, where the transmissions to/from multiple base stations are combined to give a *macro-diversity*.

Here we consider the effects of path loss and shadowing and ignore multipath-fading. Suppose that the received signal power corresponding to the link with the ith base-station, Ω_{p_i}, has the probability density function

$$p_{\Omega_{p_i\ (\mathrm{dBm})}}(x) = \frac{1}{\sqrt{2\pi}\sigma_\Omega} \exp\left\{ -\frac{(x-\mu_{\Omega_{p_i\ (\mathrm{dBm})}})^2}{2\sigma_\Omega^2} \right\} .$$

where

$$\mu_{\Omega_{p_i\ (\mathrm{dBm})}} = \mathrm{E}[\Omega_{p_i\ (\mathrm{dBm})}]$$

The Ω_{p_i} are assumed to be statistically independent.

a) 5 marks: The reverse link uses selection combining such that the best base-station is always selected. In this case,

$$\Omega^{\mathrm{s}}_{p\ (\mathrm{dBm})} = \max\left\{\Omega_{p_1\ (\mathrm{dBm})}, \ldots, \Omega_{p_L\ (\mathrm{dBm})}\right\}$$

An outage occurs if $\Omega^{\mathrm{s}}_{p\ (\mathrm{dBm})} \leq \Omega_{\mathrm{th}\ (\mathrm{dBm})}$. What is the probability of outage?

b) 5 marks: The forward link uses coherent combining such that

$$\Omega^{\mathrm{mr}}_{p\ (\mathrm{dBm})} = \Omega_{p_1\ (\mathrm{dBm})} + \ldots + \Omega_{p_L\ (\mathrm{dBm})}$$

Again, an outage occurs if $\Omega^{\mathrm{mr}}_{p\ (\mathrm{dBm})} \leq \Omega_{\mathrm{th}\ (\mathrm{dBm})}$. What is the probability of outage if

$$\mu_{\Omega_{p_1\ (\mathrm{dBm})}} = \mu_{\Omega_{p_2\ (\mathrm{dBm})}} = \cdots = \mu_{\Omega_{p_L\ (\mathrm{dBm})}}?$$

4) Consider the reception of a signal in the presence of a single co-channel interferer and neglect the effect of AWGN. The received signal power, C, and interference power, I, due to Rayleigh fading have the exponential distributions

$$\begin{aligned} p_C(x) &= \frac{1}{\bar{C}} e^{-x/\bar{C}} \\ p_I(y) &= \frac{1}{\bar{I}} e^{-x/\bar{I}} \end{aligned}$$

where $\bar{C}$ and $\bar{I}$ are the average received signal power and interference power, respectively.

a) 5 marks: Assuming that C and I are independent random variables, find the probability density function for the carrier-to-interference ratio

$$\lambda = \frac{C}{I} \ .$$

Hint: If X and Y are independent random variables, then the probability density function of $U = X/Y$ is

$$p_U(u) = \int p_{XY}(v, v/u)|v/u^2|dv \ .$$

b) 5 marks: Now suppose that the system uses 2-branch selection diversity. The branches are independent and balanced (i.e., the distribution $p_U(u)$ is the same for each branch. What is the probability density function of λ at the output of the selective combiner?

5) Suppose that a system uses selection diversity. The branches experience independent Rayleigh fading. However, the average received bit energy-to-noise ratio on each diversity branch is different, such that

$$\bar{\gamma}_i = 2^{-i}\gamma_o \quad i = 1, \ldots, L$$

a) 5 marks: Find the probability density function of the bit energy-to-noise ratio at the output of the selective combiner, denoted by γ_b^s.

b) 5 marks: If DPSK modulation is used, write down an expression for the probability of bit error. Obtain a closed-form expression if possible; otherwise leave your expression in integral form.