Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Final Exam

Fall 2010

Wednesday December 15, 2:50pm - 5:40pm

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

1) Consider the following time-invariant channel model

$$g(t,\tau) = \frac{1}{\sqrt{L}} \sum_{k=1}^{L} \delta(\tau - (k-1)\Delta_\tau)$$

where Δ_τ is the uniform spacing between equal strength channel taps.

a) 6 marks: Find the magnitude and phase response of the channel, i.e., $|T(t,f)|$ and $\angle T(t,f)$, where $T(t,f)$ is the Fourier transform of $g(t,\tau)$. Simplify as much as possible.

b) 4 marks: What is the mean delay μ_τ and rms delay spread σ_τ of this channel? *Hint:*

$$\begin{aligned}
\sum_{k=1}^{n} k &= \frac{n(n+1)}{2} \\
\sum_{k=1}^{n} k^2 &= \frac{n(n+1)(2n+1)}{6} \\
\sum_{k=0}^{n} k^3 &= \frac{n^2(n+1)^2}{4}
\end{aligned}$$

2) Suppose that a mobile station is at distance 2 km from its serving base station and at distances 4 km and 5 km from two co-channel base stations. The path loss is described by the model

$$\mu_{\Omega_p\ (\mathrm{dBm})}(d) = \mu_{\Omega_p\ (\mathrm{dBm})}(d_o) - 10\beta \log_{10}\left(\frac{d}{d_o}\right)$$

where $\mu_{\Omega_p\ (\mathrm{dBm})}(d_o) = -70$ dBm, $d_o = 1.6$ km, and $\beta = 3.68$. Each link experiences independent log-normal shadowing with shadow standard deviation $\sigma_\Omega = 8$ dB.

a) 5 marks: Using the Fenton-Wilkinson method find the proability density function of the total interfering power in decibel units.

b) 3 marks: What is the carrier-to-interference ratio in decibel units?

c) 2 marks: If the carrier-to-interference ratio must be greater than 10 dB, what is the probability of outage?

3) The squared Euclidean distance between a pair of CPM band-pass waveforms, $s(t; \mathbf{x}^{(i)})$ and $s(t; \mathbf{x}^{(j)})$, is

$$D^2 = \int_0^\infty \left[s(t; \mathbf{x}^{(i)}) - s(t; \mathbf{x}^{(j)}) \right]^2 \mathrm{d}t \ .$$

a) 6 marks: By using trigonometric manipulations show that

$$D^2 = 2E \frac{1}{T} \int_0^\infty \left[1 - \cos \Delta_\phi(t) \right] \mathrm{d}t \ ,$$

where E is the energy per modulated symbol, T is the symbol period and $\Delta_\phi(t)$ is the (excess) phase difference between the two waveforms.

b) 4 marks: What is the *minimum* squared Euclidean distance between a pair of MSK waveforms?

4) OFDM systems are known to be resilient to timing errors. Suppose that the OFDM complex envelope

$$\tilde{s}(t) = A \sum_{n=0}^{N-1} x_n \exp\left\{ j \frac{2\pi n t}{T} \right\}$$

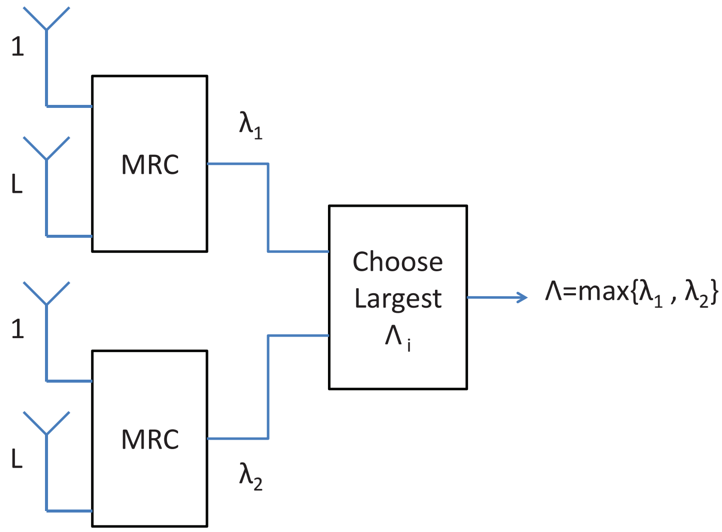
is sampled at time instants $t = kT_s + \Delta_t$, where Δ_t is a timing offset, to yield the samples

$$R_k = \tilde{s}(kT_s + \Delta_t), k = 0, \ldots, N-1$$

and an N-point FFT is taken on the samples $\{R_k\}_{k=0}^{N-1}$ to yield the coefficients $\{Z_n\}_{n=0}^{N-1}$.

a) 10 marks: Assume that the timing offset Δ_t lies somewhere in the OFDM guard interval such that all N FFT coefficients belong to the same OFDM block. Determine the FFT coefficients $\{Z_n\}_{n=0}^{N-1}$.

5) Consider a system that employs 2-branch selection diversity, where each diversity branch consists of L antennas with maximal ratio combining as shown below.



Assume

a) 5 marks:

b) 5 marks: