Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Final Exam

Fall 2011

Friday December 16, 2:50pm - 5:40pm

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

1) The power delay profile for a WSSUS channel is given by

$$\phi_{gg}(\tau) = \begin{cases} 0.5[1 + \cos(2\pi\tau/T)] \ , & 0 \leq \tau \leq T/2 \\ 0 \ , & \text{otherwise} \end{cases}$$

a) 3 marks: Find the channel frequency correlation function.

b) 4 marks: Calculate the mean delay and rms delay spread.

c) 3 marks: If $T = 0.1$ ms, determine whether the channel exhibits frequency-selective fading to the GSM system.

2) One simple model for shadow simulation is to model log-normal shadowing as a Gaussian white noise process that is filtered by a first-order low-pass filter. With this model

$$\Omega_{k+1\ (\mathrm{dBm})} = \zeta \Omega_{k\ (\mathrm{dBm})} + (1-\zeta) v_k \ ,$$

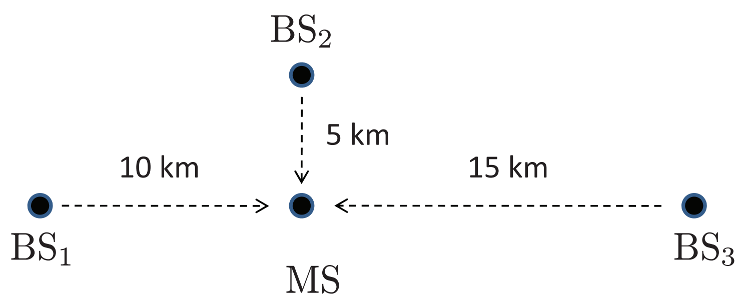
where $\Omega_{k\ (\mathrm{dBm})}$ is the mean envelope or mean squared-envelope, expressed in dBm units, that is experienced at *spatial* index k, ζ is a parameter that controls the spatial correlation of the shadows, and v_k is a zero-mean Gaussian random variable with $\phi_{vv}(n) = \tilde{\sigma}^2 \delta(n)$.

a) 6 marks: Show that the resulting spatial autocorrelation function of $\Omega_{k\ (\mathrm{dBm})}$ is

$$\phi_{\Omega_{(\mathrm{dBm})}\Omega_{(\mathrm{dBm})}}(n) = \frac{1-\zeta}{1+\zeta} \tilde{\sigma}^2 \zeta^{|n|} \ .$$

b) 4 marks: What is the mean and variance of $\Omega_{k\ (\mathrm{dBm})}$ at any spatial index k?

3) Consider the simulcast system shown in the figure below. A mobile station lies at a distance of 5 km, 10 km and 15 km from three base stations. The simulcast system is GPS synchronized so that the same waveform is transmitted from the three base stations beginning at exactly the same time epoch.



Neglect shadowing and fading, and assume that the propagation path loss follows the model

$$\mu_{\Omega_p \text{ (dBm)}}(d) = \mu_{\Omega_p \text{ (dBm)}}(d_o) - 10\beta \log_{10}(d/d_o) \quad \text{(dBm)}$$

where $\beta = 3.5$, and $\mu_{\Omega_p \text{ (dBm)}}(d_o) = 1$ microwatt at $d_o = 1$ km.

a) 5 marks: Plot the effective power delay profile that the mobile station will observe.

b) 5 marks: What is the mean delay and rms delay spread.

4) An OFDM signal with a large number of sub-carriers N and no guard interval ($G = 0$) has a complex envelope that can be approximated as a zero-mean complex Gaussian random process. Assume an "ideal" OFDM signal spectrum, where the power spectrum of the complex low-pass envelope is

$$S_{\tilde{s}\tilde{s}}(f) = \begin{cases} S_0 & , \quad |f| \leq 1/2T_s \\ 0 & , \quad \text{elsewhere} \end{cases}$$

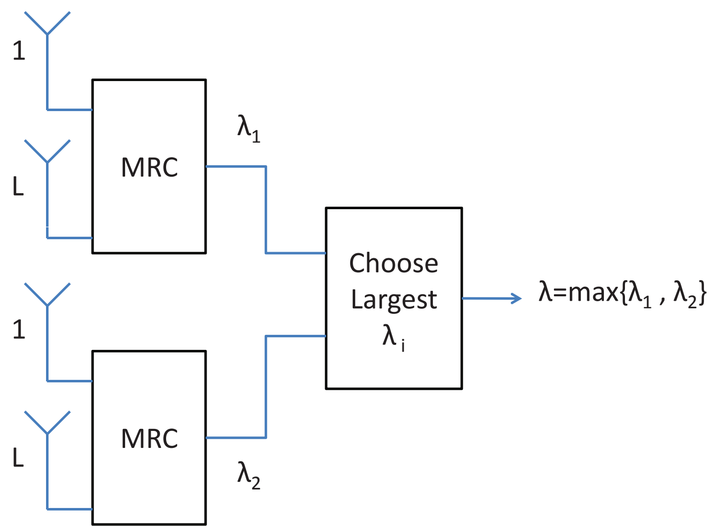
where $T = NT_s$. Note that the complex envelope $\tilde{s}(t)$ is a random process that is very similar to a complex fading envelope $g(t)$ with an associated Doppler spectrum $S_{gg}(f)$.

a) Using the above power spectrum, what is the probability density function of the magnitude of the complex envelope, $|\tilde{s}(t)|$, at any time t.

b) Suppose that the RF power amplifier will clip the OFDM waveform if the magnitude of the complex envelope $|\tilde{s}(t)|$ exceeds the level ΘR_{rms}, where R_{rms} is the rms envelope level $\sqrt{E[|\tilde{s}(t)|^2]}$. What is the probability that the OFDM waveform will be clipped at any time t?

c) Suppose that a continuous stream of OFDM symbols is transmitted. How many times per second on average will the OFDM waveform be clipped?

5) Consider a system that employs 2-branch selection diversity, where each diversity branch consists of L antennas with maximal ratio combining as shown below. Assume that all input MRC diversity branches are equal, i.e., $\bar{\gamma}_{ij} = \bar{\gamma}_c$, $i = 1, 2$, $j = 1, \ldots, L$.



a) 5 marks: Derive an expression for the cumulative distribution function of the symbol energy-to-noise ratio, λ, at the output of the selective combiner.

b) 3 marks: Derive an expression for the probability density function of the symbol energy-to-noise ratio, λ, at the output of the selective combiner.

c) 2 marks: Write down an integral expression for the probability of bit error with BPSK modulation. You do not have to solve the integral!