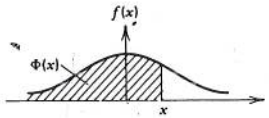
APPENDIX D



*Tabulation of the Standard Normal Distribution**

x	Φ(x)	f(x)	x	Φ(x)	f(x)	x	Φ(x)	f(x)
.00	.50000	.39894	.50	.69146	.35207	1.00	.84134	.24197
.01	.50399	.39892	.51	.69497	.35029	1.01	.84375	.23955
.02	.50798	.39886	.52	.69847	.34849	1.02	.84614	.23713
.03	.51197	.39876	.53	.70194	.34667	1.03	.84850	.23471
.04	.51595	.39862	.54	.70540	.34482	1.04	.85083	.23230
.05	.51994	.39844	.55	.70884	.34294	1.05	.85314	.22988
.06	.52392	.39822	.56	.71226	.34105	1.06	.85543	.22747
.07	.52790	.39797	.57	.71566	.33912	1.07	.85769	.22506
.08	.53188	.39767	.58	.71904	.33718	1.08	.85993	.22265
.09	.53586	.39733	.59	.72240	.33521	1.09	.86214	.22025
.10	.53983	.39695	.60	.72575	.33322	1.10	.86433	.21785
.11	.54380	.39654	.61	.72907	.33121	1.11	.86650	.21546
.12	.54776	.39608	.62	.73237	.32918	1.12	.86864	.21307
.13	.55172	.39559	.63	.73565	.32713	1.13	.87076	.21069
.14	.55567	.39505	.64	.73891	.32506	1.14	.87286	.20831
.15	.55982	.39448	.65	.74215	.32297	1.15	.87493	.20594
.16	.56356	.39387	.66	.74537	.32086	1.16	.87698	.20357
.17	.56750	.39322	.67	.74857	.31874	1.17	.87900	.20121
.18	.57142	.39253	.68	.75175	.31659	1.18	.88100	.19886
.19	.57535	.39181	.69	.75490	.31443	1.19	.88298	.19652
.20	.57926	.39104	.70	.75804	.31225	1.20	.88493	.19419
.21	.58317	.39024	.71	.76115	.31006	1.21	.88686	.19186
.22	.58706	.38940	.72	.76424	.30785	1.22	.88877	.18954
.23	.59095	.38853	.73	.76730	.30563	1.23	.89065	.18724
.24	.59484	.38762	.74	.77035	.30339	1.24	.89251	.18494
.25	.59871	.38667	.75	.77337	.30114	1.25	.89435	.18265
.26	.60257	.38568	.76	.77637	.29887	1.26	.89617	.18037
.27	.60643	.38466	.77	.77935	.29659	1.27	.89796	.17810
.28	.61026	.38361	.78	.78230	.29431	1.28	.89973	.17585
.29	.61409	.38251	.79	.78524	.29200	1.29	.90147	.17360
.30	.61791	.38139	.80	.78814	.28969	1.30	.90320	.17137
.31	.62172	.38023	.81	.79103	.28737	1.31	.90490	.16915
.32	.62552	.37903	.82	.79389	.28504	1.32	.90658	.16694
.33	.62930	.37780	.83	.79673	.28269	1.33	.90824	.16474
.34	.63307	.37654	.84	.79955	.28034	1.34	.90988	.16256
.35	.63683	.37524	.85	.80234	.27798	1.35	.91149	.16038
.36	.64058	.37391	.86	.80511	.27562	1.36	.91308	.15822
.37	.64431	.37255	.87	.80785	.27324	1.37	.91466	.15608
.38	.64803	.37115	.88	.81057	.27086	1.38	.91621	.15395
.39	.65173	.36973	.89	.81327	.26848	1.39	.91774	.15183
.40	.65542	.36827	.90	.81594	.26609	1.40	.91924	.14973
.41	.65910	.36678	.91	.81859	.26369	1.41	.92073	.14764
.42	.66276	.36526	.92	.82121	.26129	1.42	.92220	.14556
.43	.66640	.36371	.93	.82381	.25888	1.43	.92364	.14350
.44	.67003	.36213	.94	.82639	.25647	1.44	.92507	.14146
.45	.67365	.36053	.95	.82894	.25406	1.45	.92647	.13943
.46	.67724	.35889	.96	.83147	.25164	1.46	.92786	.13742
.47	.68082	.35723	.97	.83398	.24923	1.47	.92922	.13542
.48	.68439	.35553	.98	.83646	.24681	1.48	.93056	.13344
.49	.68793	.35381	.99	.83891	.24439	1.49	.93189	.13147
.50	.69146	.35207	1.00	.84134	.24197	1.50	.93319	.12952

* Abridged from *Biometrika Tables for Statisticians*, vol. 1 (2nd edition), edited by E. S. Pearson and H. O. Hartley, Cambridge University Press, London, 1958, table 1, with permission of the Biometrika Trustees.

x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$
1.50	.93319	.12952	2.00	.97725	.05399	2.50	.99379	.01753
1.51	.93448	.12758	2.01	.97778	.05292	2.51	.99396	.01709
1.52	.93574	.12566	2.02	.97831	.05186	2.52	.99413	.01667
1.53	.93699	.12376	2.03	.97882	.05082	2.53	.99430	.01625
1.54	.93822	.12188	2.04	.97932	.04980	2.54	.99446	.01585
1.55	.93943	.12001	2.05	.97982	.04879	2.55	.99461	.01545
1.56	.94062	.11816	2.06	.98030	.04780	2.56	.99477	.01506
1.57	.94179	.11632	2.07	.98077	.04682	2.57	.99492	.01468
1.58	.94295	.11450	2.08	.98124	.04586	2.58	.99506	.01431
1.59	.94408	.11270	2.09	.98169	.04491	2.59	.99520	.01394
1.60	.94520	.11092	2.10	.98214	.04398	2.60	.99534	.01358
1.61	.94630	.10915	2.11	.98257	.04307	2.61	.99547	.01323
1.62	.94738	.10741	2.12	.98300	.04217	2.62	.99560	.01289
1.63	.94845	.10567	2.13	.98341	.04128	2.63	.99573	.01256
1.64	.94950	.10396	2.14	.98382	.04041	2.64	.99585	.01223
1.65	.95053	.10226	2.15	.98422	.03955	2.65	.99598	.01191
1.66	.95154	.10059	2.16	.98461	.03871	2.66	.99609	.01160
1.67	.95254	.09893	2.17	.98500	.03788	2.67	.99621	.01130
1.68	.95352	.09728	2.18	.98537	.03706	2.68	.99632	.01100
1.69	.95449	.09566	2.19	.98574	.03626	2.69	.99643	.01071
1.70	.95543	.09405	2.20	.98610	.03547	2.70	.99653	.01042
1.71	.95637	.09246	2.21	.98645	.03470	2.71	.99664	.01014
1.72	.95728	.09089	2.22	.98679	.03394	2.72	.99674	.00987
1.73	.95818	.08933	2.23	.98713	.03319	2.73	.99683	.00961
1.74	.95907	.08780	2.24	.98745	.03246	2.74	.99693	.00935
1.75	.95994	.08628	2.25	.98778	.03174	2.75	.99702	.00909
1.76	.96080	.08478	2.26	.98809	.03103	2.76	.99711	.00885
1.77	.96164	.08329	2.27	.98840	.03034	2.77	.99720	.00861
1.78	.96246	.08183	2.28	.98870	.02965	2.78	.99728	.00837
1.79	.96327	.08038	2.29	.98899	.02898	2.79	.99736	.00814
1.80	.96407	.07895	2.30	.98928	.02833	2.80	.99744	.00792
1.81	.96485	.07754	2.31	.98956	.02768	2.81	.99752	.00770
1.82	.96562	.07614	2.32	.98983	.02705	2.82	.99760	.00748
1.83	.96638	.07477	2.33	.99010	.02643	2.83	.99767	.00727
1.84	.96712	.07341	2.34	.99036	.02582	2.84	.99774	.00707
1.85	.96784	.07206	2.35	.99061	.02522	2.85	.99781	.00687
1.86	.96856	.07074	2.36	.99086	.02463	2.86	.99788	.00668
1.87	.96926	.06943	2.37	.99111	.02406	2.87	.99795	.00649
1.88	.96995	.06814	2.38	.99134	.02349	2.88	.99801	.00631
1.89	.97062	.06687	2.39	.99158	.02294	2.89	.99807	.00613
1.90	.97128	.06562	2.40	.99180	.02239	2.90	.99813	.00595
1.91	.97193	.06438	2.41	.99202	.02186	2.91	.99819	.00578
1.92	.97257	.06316	2.42	.99224	.02134	2.92	.99825	.00562
1.93	.97320	.06195	2.43	.99245	.02083	2.93	.99831	.00545
1.94	.97381	.06077	2.44	.99266	.02033	2.94	.99836	.00530
1.95	.97441	.05959	2.45	.99286	.01984	2.95	.99841	.00514
1.96	.97500	.05844	2.46	.99305	.01936	2.96	.99846	.00499
1.97	.97558	.05730	2.47	.99324	.01889	2.97	.99851	.00485
1.98	.97615	.05618	2.48	.99343	.01842	2.98	.99856	.00471
1.99	.97670	.05508	2.49	.99361	.01797	2.99	.99861	.00457
2.00	.97725	.05399	2.50	.99379	.01753	3.00	.99865	.00443

x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$
3.00	.99865	.00443	3.50	.99977	.00087	4.00	.99997	.00013
3.01	.99869	.00430	3.51	.99978	.00084	4.01	.99997	.00013
3.02	.99874	.00417	3.52	.99978	.00081	4.02	.99997	.00012
3.03	.99878	.00405	3.53	.99979	.00079	4.03	.99997	.00012
3.04	.99882	.00393	3.54	.99980	.00076	4.04	.99997	.00011
3.05	.99886	.00381	3.55	.99981	.00073	4.05	.99997	.00011
3.06	.99889	.00370	3.56	.99981	.00071	4.06	.99998	.00011
3.07	.99893	.00358	3.57	.99982	.00068	4.07	.99998	.00010
3.08	.99897	.00348	3.58	.99983	.00066	4.08	.99998	.00010
3.09	.99900	.00337	3.59	.99983	.00063	4.09	.99998	.00009
3.10	.99903	.00327	3.60	.99984	.00061	4.10	.99998	.00009
3.11	.99906	.00317	3.61	.99985	.00059	4.11	.99998	.00009
3.12	.99910	.00307	3.62	.99985	.00057	4.12	.99998	.00008
3.13	.99913	.00298	3.63	.99986	.00055	4.13	.99998	.00008
3.14	.99916	.00288	3.64	.99986	.00053	4.14	.99998	.00008
3.15	.99918	.00279	3.65	.99987	.00051	4.15	.99998	.00007
3.16	.99921	.00271	3.66	.99987	.00049	4.16	.99998	.00007
3.17	.99924	.00262	3.67	.99988	.00047	4.17	.99998	.00007
3.18	.99926	.00254	3.68	.99988	.00046	4.18	.99999	.00006
3.19	.99929	.00246	3.69	.99989	.00044	4.19	.99999	.00006
3.20	.99931	.00238	3.70	.99989	.00042	4.20	.99999	.00006
3.21	.99934	.00231	3.71	.99990	.00041	4.21	.99999	.00006
3.22	.99936	.00224	3.72	.99990	.00039	4.22	.99999	.00005
3.23	.99938	.00216	3.73	.99990	.00038	4.23	.99999	.00005
3.24	.99940	.00210	3.74	.99991	.00037	4.24	.99999	.00005
3.25	.99942	.00203	3.75	.99991	.00035	4.25	.99999	.00005
3.26	.99944	.00196	3.76	.99992	.00034	4.26	.99999	.00005
3.27	.99946	.00190	3.77	.99992	.00033	4.27	.99999	.00004
3.28	.99948	.00184	3.78	.99992	.00031	4.28	.99999	.00004
3.29	.99950	.00178	3.79	.99992	.00030	4.29	.99999	.00004
3.30	.99952	.00172	3.80	.99993	.00029	4.30	.99999	.00004
3.31	.99953	.00167	3.81	.99993	.00028	4.31	.99999	.00004
3.32	.99955	.00161	3.82	.99993	.00027	4.32	.99999	.00004
3.33	.99957	.00156	3.83	.99994	.00026	4.33	.99999	.00003
3.34	.99958	.00151	3.84	.99994	.00025	4.34	.99999	.00003
3.35	.99960	.00146	3.85	.99994	.00024	4.35	.99999	.00003
3.36	.99961	.00141	3.86	.99994	.00023	4.36	.99999	.00003
3.37	.99962	.00136	3.87	.99995	.00022	4.37	.99999	.00003
3.38	.99964	.00132	3.88	.99995	.00021	4.38	.99999	.00003
3.39	.99965	.00127	3.89	.99995	.00021	4.39	.99999	.00003
3.40	.99966	.00123	3.90	.99995	.00020	4.40	.99999	.00002
3.41	.99968	.00119	3.91	.99995	.00019	4.41	.99999	.00002
3.42	.99969	.00115	3.92	.99996	.00018	4.42	1.00000	.00002
3.43	.99970	.00111	3.93	.99996	.00018	4.43	1.00000	.00002
3.44	.99971	.00107	3.94	.99996	.00017	4.44	1.00000	.00002
3.45	.99972	.00104	3.95	.99996	.00016	4.45	1.00000	.00002
3.46	.99973	.00100	3.96	.99996	.00016	4.46	1.00000	.00002
3.47	.99974	.00097	3.97	.99996	.00015	4.47	1.00000	.00002
3.48	.99975	.00094	3.98	.99997	.00014	4.48	1.00000	.00002
3.49	.99976	.00090	3.99	.99997	.00014	4.49	1.00000	.00002
3.50	.99977	.00087	4.00	.99997	.00013	4.50	1.00000	.00002

Pythagorean relations

$$\sin^2\alpha + \cos^2\alpha = 1, \qquad 1 + \tan^2\alpha = \sec^2\alpha, \qquad 1 + \cot^2\alpha = \csc^2\alpha$$

Angle-sum and angle-difference relations

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$
$$\cot(\alpha + \beta) = \frac{\cot\beta\cot\alpha - 1}{\cot\beta + \cot\alpha}$$
$$\cot(\alpha - \beta) = \frac{\cot\beta\cot\alpha + 1}{\cot\beta - \cot\alpha}$$
$$\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$$
$$\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha$$

Double-angle relations

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}, \qquad \cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$$

Multiple-angle relations

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$
$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$
$$\sin 4\alpha = 4\sin\alpha\cos\alpha - 8\sin^3\alpha\cos\alpha$$
$$\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$$
$$\sin 5\alpha = 5\sin\alpha - 20\sin^3\alpha + 16\sin^5\alpha$$
$$\cos 5\alpha = 16\cos^5\alpha - 20\cos^3\alpha + 5\cos\alpha$$
$$\sin 6\alpha = 32\cos^5\alpha\sin\alpha - 32\cos^3\alpha\sin\alpha + 6\cos\alpha\sin\alpha$$
$$\cos 6\alpha = 32\cos^6\alpha - 48\cos^4\alpha + 18\cos^2\alpha - 1$$
$$\sin n\alpha = 2\sin(n-1)\alpha\cos\alpha - \sin(n-2)\alpha$$
$$\cos n\alpha = 2\cos(n-1)\alpha\cos\alpha - \cos(n-2)\alpha$$
$$\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$$
$$\tan 4\alpha = \frac{4\tan\alpha - 4\tan^3\alpha}{1 - 6\tan^2\alpha + \tan^4\alpha}$$
$$\tan n\alpha = \frac{\tan(n-1)\alpha + \tan\alpha}{1 - \tan(n-1)\alpha\tan\alpha}$$

Formulas for Use in Trigonometry

Function-product relations

$$\sin\alpha\sin\beta = \tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha + \beta) + \tfrac{1}{2}\sin(\alpha - \beta)$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\sin(\alpha + \beta) - \tfrac{1}{2}\sin(\alpha - \beta)$$

Function-sum and function-difference relations

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$
$$\sin\alpha - \sin\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$
$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$
$$\cos\alpha - \cos\beta = -2\sin\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$
$$\tan\alpha + \tan\beta = \frac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta}, \qquad \tan\alpha - \tan\beta = \frac{\sin(\alpha - \beta)}{\cos\alpha\cos\beta}$$
$$\cot\alpha + \cot\beta = \frac{\sin(\alpha + \beta)}{\sin\alpha\sin\beta}, \qquad \cot\alpha - \cot\beta = \frac{\sin(\beta - \alpha)}{\sin\alpha\sin\beta}$$
$$\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{\tan\tfrac{1}{2}(\alpha + \beta)}{\tan\tfrac{1}{2}(\alpha - \beta)}, \qquad \frac{\sin\alpha + \sin\beta}{\cos\alpha - \cos\beta} = \cot\tfrac{1}{2}(\beta - \alpha)$$
$$\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \tan\tfrac{1}{2}(\alpha + \beta), \qquad \frac{\sin\alpha - \sin\beta}{\cos\alpha + \cos\beta} = \tan\tfrac{1}{2}(\alpha - \beta)$$

Half-angle relations

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}, \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}} = \frac{1 - \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 + \cos\alpha}$$
$$\cot\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{1 - \cos\alpha}} = \frac{1 + \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 - \cos\alpha}$$

Power relations

$$\sin^2\alpha = \tfrac{1}{2}(1 - \cos 2\alpha), \qquad \sin^3\alpha = \tfrac{1}{4}(3\sin\alpha - \sin 3\alpha)$$
$$\sin^4\alpha = \tfrac{1}{8}(3 - 4\cos 2\alpha + \cos 4\alpha)$$
$$\cos^2\alpha = \tfrac{1}{2}(1 + \cos 2\alpha), \qquad \cos^3\alpha = \tfrac{1}{4}(3\cos\alpha + \cos 3\alpha)$$
$$\cos^4\alpha = \tfrac{1}{8}(3 + 4\cos 2\alpha + \cos 4\alpha)$$
$$\tan^2\alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}, \qquad \cot^2\alpha = \frac{1 + \cos 2\alpha}{1 - \cos 2\alpha}$$

Exponential relations (α in radians), Euler's equation

$$e^{i\alpha} = \cos\alpha + i\sin\alpha, \qquad i = \sqrt{-1}$$
$$\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}, \qquad \cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$
$$\tan\alpha = -i\left(\frac{e^{i\alpha} - e^{-i\alpha}}{e^{i\alpha} + e^{-i\alpha}}\right) = -i\left(\frac{e^{2i\alpha} - 1}{e^{2i\alpha} + 1}\right)$$

FOURIER TRANSFORM PAIRS

Pair Number	$x(t)$	$X(f)$
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}(\tau f)$
2.	$2W\,\mathrm{sinc}(2Wt)$	$\Pi\left(\frac{f}{2W}\right)$
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}^2(\tau f)$
4.	$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$
5.	$t\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$
6.	$\exp(-\alpha\lvert t\rvert),\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$
7.	$\exp(-\alpha t^2)$	$\sqrt{\frac{\pi}{\alpha}}\exp\left(-\frac{\pi^2 f^2}{\alpha}\right)$
8.	$\delta(t)$	1
9.	1	$\delta(f)$
10.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
11.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$
12.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
13.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$
14.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$
15.	$\mathrm{sgn}(t)$	$(j\pi f)^{-1}$
16.	$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$
17.	$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\lambda)}{t - \lambda}\,d\lambda$	$-j\,\mathrm{sgn}(f)X(f)$
18.	$\sum_{m=-\infty}^{\infty}\delta(t - mT_s)$	$f_s\sum_{m=-\infty}^{\infty}\delta(f - mf_s), f_s = T_s^{-1}$

Note: $\mathrm{sinc}\,u = \frac{\sin \pi u}{\pi u}$

$$\Pi(u) = \begin{cases} 1, & |u| \le 1/2 \\ 0, & \text{otherwise} \end{cases}$$

$$\Lambda(u) = \begin{cases} 1 - |u|, & |u| \le | \\ 0, & \text{otherwise} \end{cases}$$

FOURIER TRANSFORM THEOREMS

Name of Theorem	Signal	Transform
1. Superposition (a_1 and a_2 arbitrary constants)	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(f) + a_2X_2(f)$
2. Time delay	$x(t - t_0)$	$X(f) \exp(-j\,2\pi f t_0)$
3a. Scale change	$x(at)$	$\lvert a\rvert^{-1}X\left(\frac{f}{a}\right)$
3b. Time reversal[1]	$x(-t)$	$X(-f) = X^*(f)$
4. Duality	$X(t)$	$x(-f)$
5a. Frequency translation	$x(t) \exp(j2\pi f_0 t)$	$X(f - f_0)$
5b. Modulation	$x(t) \cos 2\pi f_0 t$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
6. Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
7. Integration	$\int_{-\infty}^{t} x(t')\,dt'$	$(j2\pi f)^{-1}X(f) + \frac{1}{2}X(0)\delta(f)$
8. Convolution	$\int_{-\infty}^{\infty} x_1(t - t')x_2(t')\,dt'$ $= \int_{-\infty}^{\infty} x_1(t')x_2(t - t')\,dt'$	$X_1(f)X_2(f)$
9. Multiplication	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty} X_1(f - f')X_2(f')\,df'$ $= \int_{-\infty}^{\infty} X_1(f')X_2(f - f')\,df'$

[1] $x(t)$ is assumed to be real in 3b.

Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Final Exam

Fall 2012

Monday December 10, 2:50pm - 5:40pm

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

1) Consider a linear *time-invariant* channel having the impulse response

$$g(t,\tau) = \delta(\tau) + 2\delta(\tau - \tau_1) + \delta(\tau - 2\tau_1) \ .$$

a) (**4 points**) Derive a closed-form expression for magnitude response of the channel $|T(f,t)|$ and sketch showing all important points.

b) (**2 points**) Repeat part a) for the phase response of the channel $\angle T(f,t)$.

c) (**4 points**) What is the mean delay and rms delay spread of the channel.

2) A flat Rayleigh fading signal at 5.9 GHz is received by a vehicle traveling at 80 km/hr. Assume a 2-D isotropic scattering environment.

a) (**5 points**) Determine the number of positive-going zero crossings about the rms value that occur over a 5 s interval.

b) (**3 points**) Determine the average duration of a fade below the rms level.

c) (**2 points**) Determine the level crossing rate and average fade duration at a level of 10 dB below the rms value.

3) Suppose that an uncorrelated binary data sequence is transmitted by using BPSK with a root-Gaussian amplitude shaping pulse

$$H_a(f) = \left(\tau \mathrm{e}^{-\pi(f\tau)^2}\right)^{1/2}$$

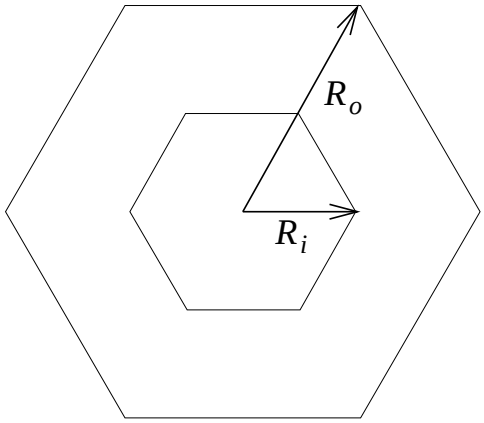
a) (**4 points**) What is the transmitted power density spectrum?

b) (**3 points**) Find the value of τ so that the power density spectrum is 20 dB below its peak value at the Nyquist frequency $1/2T$, where T is the baud duration.

c) (**2 points**) What is the corresponding time domain pulse $h_a(t)$?

d) (**1 points**) Assuming that a filter matched to $h_a(t)$ is used at the receiver, will this pulse result in intersymbol interference? Why?

4) One method for improving the capacity of a cellular system is to employ a *two-channel bandwidth* scheme, where a hexagonal cell is divided into two concentric hexagons as shown below. The inner hexagon is serviced by half-rate channels, while the outer hexagon is serviced by full-rate channels. When a mobile station crosses the boundary between the inner and outer portions of a cell a handoff occurs.



Suppose that the full-rate channels require $C/I = 7$ dB to maintain an acceptable radio link quality, while the half-rate channels require $C/I = 10$ dB.

Assume a fourth-law path loss model and suppose that the effects of envelope fading and shadowing can be ignored. Consider the reverse link and suppose that there are 6 co-channel interferers at distance D from the serving base station. It follows that the worst case C/I when a mobile station is located at distance d from the base station is $C/I = (D/d)^4/6$.

Hence, a $C/I = 7$ dB requires a 7-cell reuse cluster with $D/R_o = 4.6$, where R_o is the radius of the outer cell.

a) 2 marks: Find the required value of D/R_o so that $C/I = 7$ dB in the outer cell, where R_i is the radii of the outer cell.

a) 2 marks: Find the required value of D/R_i so that $C/I = 10$ dB in the inner cell, where R_i is the radii of the inner cell.

c) 3 marks: Use the values of D/R_i and D/R_o to determine the ratio of the inner and outer cell areas, A_i/A_o. Use the exact area of a hexagon in terms of its radius.

d) 3 marks: Let N_i and N_o be the number of channels that are allocated to the inner and outer areas of each cell, and assume that the channels are assigned such that $N_i/N_o = A_i/A_o$. Determine the increase in cell capacity (as measured in channels per cell) over a conventional *one-channel bandwidth* system that uses only full-rate channels.

5) Consider a communication link operating over a channel with propagation path loss exponent $\beta = 3.5$ and a shadow standard deviation $\sigma_\Omega = 8$ dB.

a) **4 marks:** Consider the case of two adjacent cells. A mobile station is transmitting at its maximum power and is located exactly on the cell boundary between the two base stations. In the absence of shadowing, the received power level would be equal to the receiver sensitivity, S_{RX}, i.e., the mobile station is located at distance $d_{\max}$ from each base station.

Now assume that shadowing *is* present and a soft handoff algorithm is used, such that the least attenuation link is always selected. The shadows experienced on the two possible links are independent. If an outage probability of 10% is desired for the given mobile station location (averaged over a large ensemble of realizations), what is the required margin $(M_{\mathrm{shad}} - G_{\mathrm{HO}})$, where M_{shad} is the required shadow margin for a single isolated cell and G_{HO} is the soft handoff gain?

b) **2 marks:** What is the value of G_{HO}?

c) **4 marks:** Repeat parts (a) and (b) assuming that the mobile is located on the boundaries of and equidistant from three base stations, i.e., the mobile station is located at distance $d_{\max}$ from three base stations.