Georgia Institute of Technology
School of Electrical and Computer Engineering

# ECE6604 Personal & Mobile Communications

Final Exam

Fall 2013

Monday December 9, 2:50pm - 5:40pm

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

**1)**

**a)** Consider the transmission of a bandpass signal having complex envelope $\tilde{s}(t)$ on a channel such that the received complex envelope is

$$\tilde{r}(t) = \alpha\tilde{s}(t) + \beta\tilde{s}(t - \tau_1) \ ,$$

where $\alpha$ and $\beta$ are real valued.

**i)** (**1 mark**) Find the channel impulse response $g(t, \tau)$.

**ii)** (**2 marks**) Find the channel magnitude response $|G(t, f)|$ .

**iii)** (**2 marks**) Find the channel phase response $\angle G(t, f)$.

**b)** Consider the transmission of a bandpass signal having complex envelope $\tilde{s}(t)$ on a channel such that the received complex envelope is

$$\tilde{r}(t) = \alpha\tilde{s}(t) + \beta\tilde{s}(t)\mathrm{e}^{j2\pi f_d t} \ ,$$

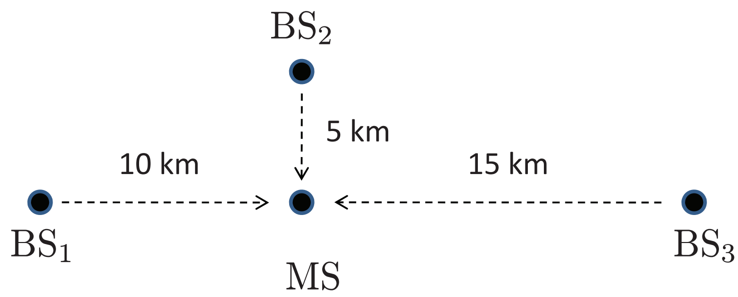
where $\alpha$ and $\beta$ are real valued.

**i)** (**1 mark**) Find the channel impulse response $g(t, \tau)$.

**ii)** (**2 marks**) Find the channel magnitude response $|G(t, f)|$.

**iii)** (**2 marks**) Find the channel phase response $\angle G(t, f)$.

**2)** Consider the system shown in the figure below. A mobile station (MS) lies at a distance of 5 km, 10 km and 15 km from three base stations, $\mathrm{BS}_i$, $i = 1, 2, 3$. $\mathrm{BS}_2$ is the serving base station, while $\mathrm{BS}_1$ and $\mathrm{BS}_3$ are co-channel base stations (co-channel interferers).




The propagation path loss follows the model

$$\mu_{\Omega_p \ (\mathrm{dBm})}(d) = \mu_{\Omega_p \ (\mathrm{dBm})}(d_o) - 10\beta \log_{10}(d/d_o) \quad (\mathrm{dBm})$$

where $\beta = 3.5$, and $\mu_{\Omega_p}(d_o) = 1$ microwatt at $d_o = 1$ km. Each radio link is affected independent log-normal shadowing with shadow standard deviation $\sigma_\Omega = 8$ dB. Ignore envelope fading.

**a) 5 marks:** Obtain the probability density function of the total interfering power observed at the MS in decibel units.

**b) 3 marks:** What is the probability density function of the carrier-to-interference ratio observed at the MS in decibel units?

**c) 2 marks:** If the carrier-to-interference ratio must be greater than 6 dB for adequate radio link performance, what is the probability of outage?

**3)** A guard interval consisting of a cyclic prefix or cyclic suffix is used in OFDM systems to mitigate the effects of channel time dispersion.

**a) 5 marks:** Assess the cost of the cyclic prefix in terms of

**i)** bandwidth and/or data rate.

**ii)** transmitter power.

**a) 5 marks:** Suppose the a guard interval of 500 ns is used. The data rate with 64-QAM modulation is 54 Mb/s. The power penalty due to the guard interval is to be kept less than 1 dB. What is the required value of $G$ (constrained to an integer) and the minimum possible OFDM block size $N$ (constrained to $2^k$ for some $k$)?

4) Consider binary, orthogonal signaling using non-coherent FSK modulation and demodulation. The probability of bit error for non-coherent FSK on an AWGN channel is

$$P_b(\gamma_b) = \frac{1}{2}e^{-\gamma_b/2}$$

where $\gamma_b = \alpha^2 E_b/N_o$ is the received bit energy-to-noise ratio. Derive the corresponding probability of bit error for

a) **5 marks:** a flat Rayleigh fading channel.

b) **5 marks:** a flat Ricean fading channel. An integral expression is acceptable, but it does reduce to closed form.

**5)** Consider a BPSK modulated system with simple repetition code and time interleaving, such that each data bit is transmitted $L$ times and each transmission experiences independent identically distributed (i.i.d.) Rayleigh fading. If symbol $\tilde{s}$ is transmitted, the corresponding $L$ correlator or matched filter outputs at the receiver are

$$\tilde{r}_k = \alpha_k \tilde{s} + \tilde{n}_k \ , \quad k = 1, \ldots, L$$

where $\tilde{s}$ is the transmitted BPSK symbol chosen from the alphabet $\{\pm\sqrt{2E}\}$, the $\alpha_k$ are i.i.d. Rayleigh random variables, and the $\tilde{n}_k$ are i.i.d. zero-mean complex Gaussian random variables with variance $\frac{1}{2}\mathrm{E}[|\tilde{n}_k|^2]=N_o$.

**a) 3 marks:** One decoding strategy is to combine the $\tilde{r}_k, \ k = 1, \ldots, L$ using maximal ratio combining and then make a bit decision. What is the probability of decision error in terms of the average received bit energy-to-noise ratio, $\bar{\gamma}_b$?

**b) 5 marks:** Another decoding strategy is to make a hard decision as to which symbol was transmitted for each of the $\tilde{r}_k, \ k = 1, \ldots, L$, and then make a majority logic decision (assuming $L$ is odd) as to which data bit was transmitted, i.e., if more of the $L$ symbols comprising each bit are decided to be $+\sqrt{2E}$ than $-\sqrt{2E}$, then choose the data bit corresponding to symbol $+\sqrt{2E}$. What is the probability of decision error in terms of the average received bit energy-to-noise ratio, $\bar{\gamma}_b$?

**c) 2 marks:** Evaluate the probability of bit error in parts a) and b) when $L = 3$ and $\bar{\gamma}_b = 20$ dB.