Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Final Exam

Fall 2014

Tuesday December 9, 8:00am - 10:50pm

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.

1) The *scattering function* for a WSSUS channel is given by

$$\psi_S(\tau,\nu) = \begin{cases} \frac{\Omega_p}{100} b\mathrm{e}^{-b\tau}, & 0 \le |\nu| \le 50 \text{ Hz}, 0 \le \tau \le \infty \\ 0, & \text{elsewhere} \end{cases}$$

a) 2 marks: What is the speed of the mobile station?

b) 2 marks: What is the channel correlation function $\psi_g(\Delta t;\tau)$?

c) 2 marks: If the faded envelope is sampled, $g_k = g(kT_s)$, what sample spacings T_s will yield uncorrelated samples, g_k?

d) 4 marks: What is the envelope level crossing rate, L_R? Your result will depend on τ.

2) Suppose that we construct a discrete-time Rayleigh fading simulator that generates the complex faded envelope $\{g_k\}$, where

$$g_k = g_{I,k} + jg_{Q,k}$$

according to a Markov process with state equation

$$\begin{aligned} g_{I,k+1} &= \zeta g_{I,k} + (1-\zeta)w_{1,k} \\ g_{Q,k+1} &= \zeta g_{Q,k} + (1-\zeta)w_{2,k} \end{aligned}$$

where $0 < \zeta < 1$, and the $w_{i,k}$ are independent identically distributed zero-mean Gaussian random variables, i.e., $\mathrm{E}[w_{i,j}w_{i,k}] = \sigma^2\delta_{jk},\ i = 1,2$, where

$$\delta_{jk} = \begin{cases} 0, & j \neq k \\ 1, & j = k \end{cases}$$

a) 7 marks: Derive the discrete autocorrelation function

$$\phi_{g_I g_I}(n) = \mathrm{E}[g_{I,k}g_{I,k+n}]$$

b) 3 marks: Derive the discrete cross-correlation function

$$\phi_{g_I g_Q}(n) = \mathrm{E}[g_{I,k}g_{Q,k+n}]$$

3) Suppose that an uncorrelated binary data sequence is transmitted by using binary PSK (BPSK) with a root-Gaussian amplitude shaping pulse

$$H_a(f) = \left(\tau \mathrm{e}^{-\pi(f\tau)^2}\right)^{1/2}$$

a) 4 marks: What is the transmitted power density spectrum?

b) 2 marks: Find the value of τ so that the power density spectrum is 40 dB below its peak value at the Nyquist frequency $1/2T$, where T is the baud period.

c) 2 marks: What is the corresponding time domain pulse $h_a(t)$?

d) 2 marks: What is the matched filter to $h_a(t)$?

4) Consider a bandpass binary modulated signal having the complex envelope

$$\tilde{s}(t) = A \sum_n x_n h_a(t - nT) \ ,$$

where $x_n \in \{-1, \ +1\}$ and $h_a(t)$ is the amplitude shaping pulse. The data sequence $\{x_n\}$ is correlated such that

$$\phi_{xx}(n) = \frac{1}{2}\mathrm{E}[x_k^* x_{k+n}] = \rho^{|n|} \ .$$

a) 10 marks: Derive the power density spectrum of the complex envelope, $S_{\tilde{s}\tilde{s}}(f)$.

5) Consider Alamouti's transmit diversity scheme. The receiver diversity combiner for 2×2 transmit diversity is given by

$$\begin{aligned}
\tilde{\mathbf{v}}_{(1)} &= g_{1,1}^{*}\tilde{\mathbf{r}}_{(1),1} + g_{2,1}\tilde{\mathbf{r}}_{(2),1}^{*} + g_{1,2}^{*}\tilde{\mathbf{r}}_{(1),2} + g_{2,2}\tilde{\mathbf{r}}_{(2),2}^{*} \\
\tilde{\mathbf{v}}_{(2)} &= g_{2,1}^{*}\tilde{\mathbf{r}}_{(1),1} - g_{1,1}\tilde{\mathbf{r}}_{(2),1}^{*} + g_{2,2}^{*}\tilde{\mathbf{r}}_{(1),2} - g_{1,2}\tilde{\mathbf{r}}_{(2),2}^{*} \ .
\end{aligned}$$

a) 10 marks: Construct the combiner for the case of $2 \times L$ diversity.