Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

Final Exam

Fall 2015

Thursday December 10, 8:00am - 10:50pm

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.
- Math tables are attached at the end of this exam.
  You do not need to turn them in.

1) Consider the reverse link of a cellular system that uses a 7-cell reuse cluster with omnidirectional base station antennas. Ignore envelope fading and shadowing, and assume the simple path loss model

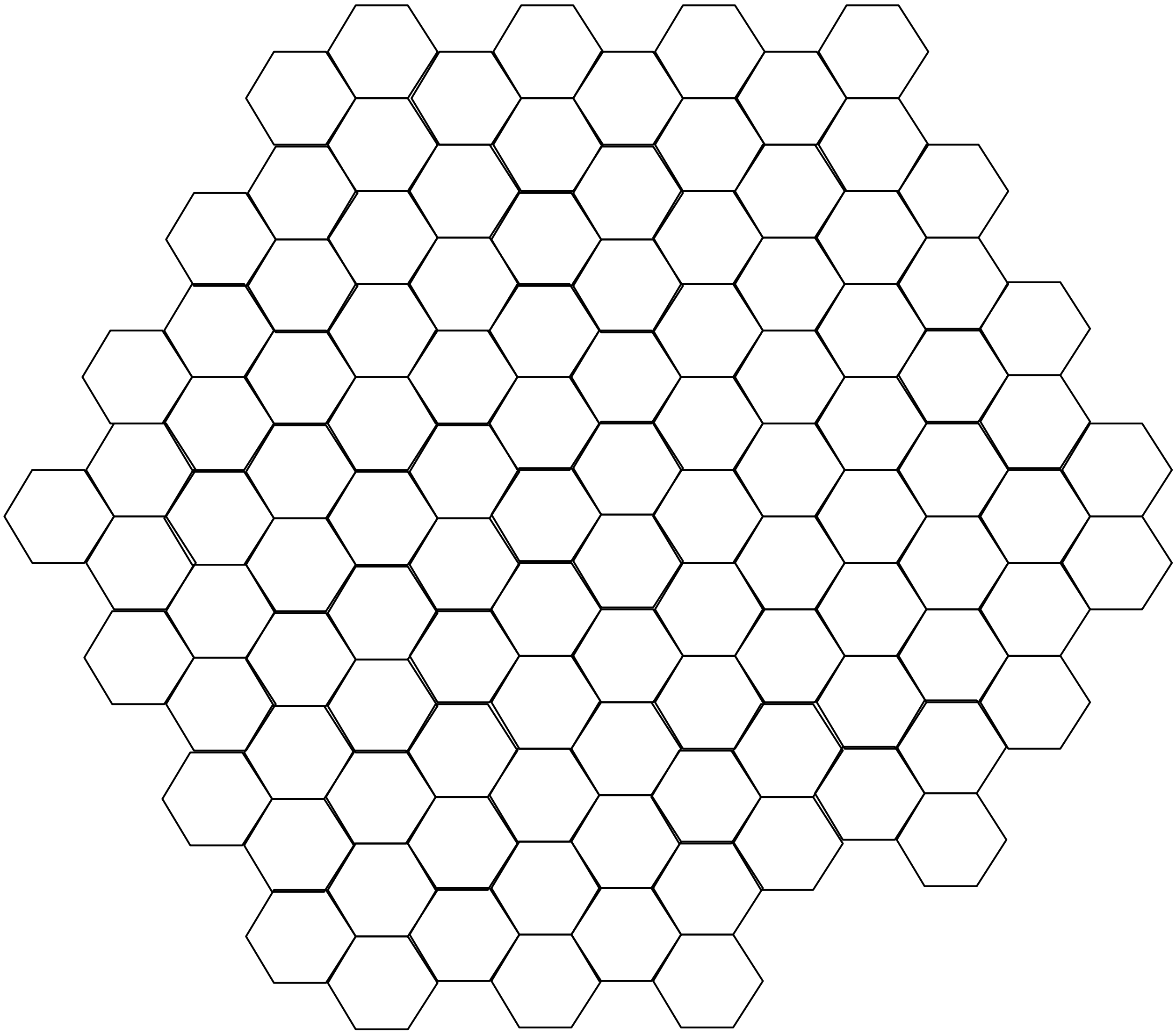
$$\mu_{\Omega_p \text{ (dBm)}}(d) = \mu_{\Omega_p \text{ (dBm)}}(d_o) - 10\beta \log_{10}(d/d_o) \quad \text{(dBm)} \ ,$$

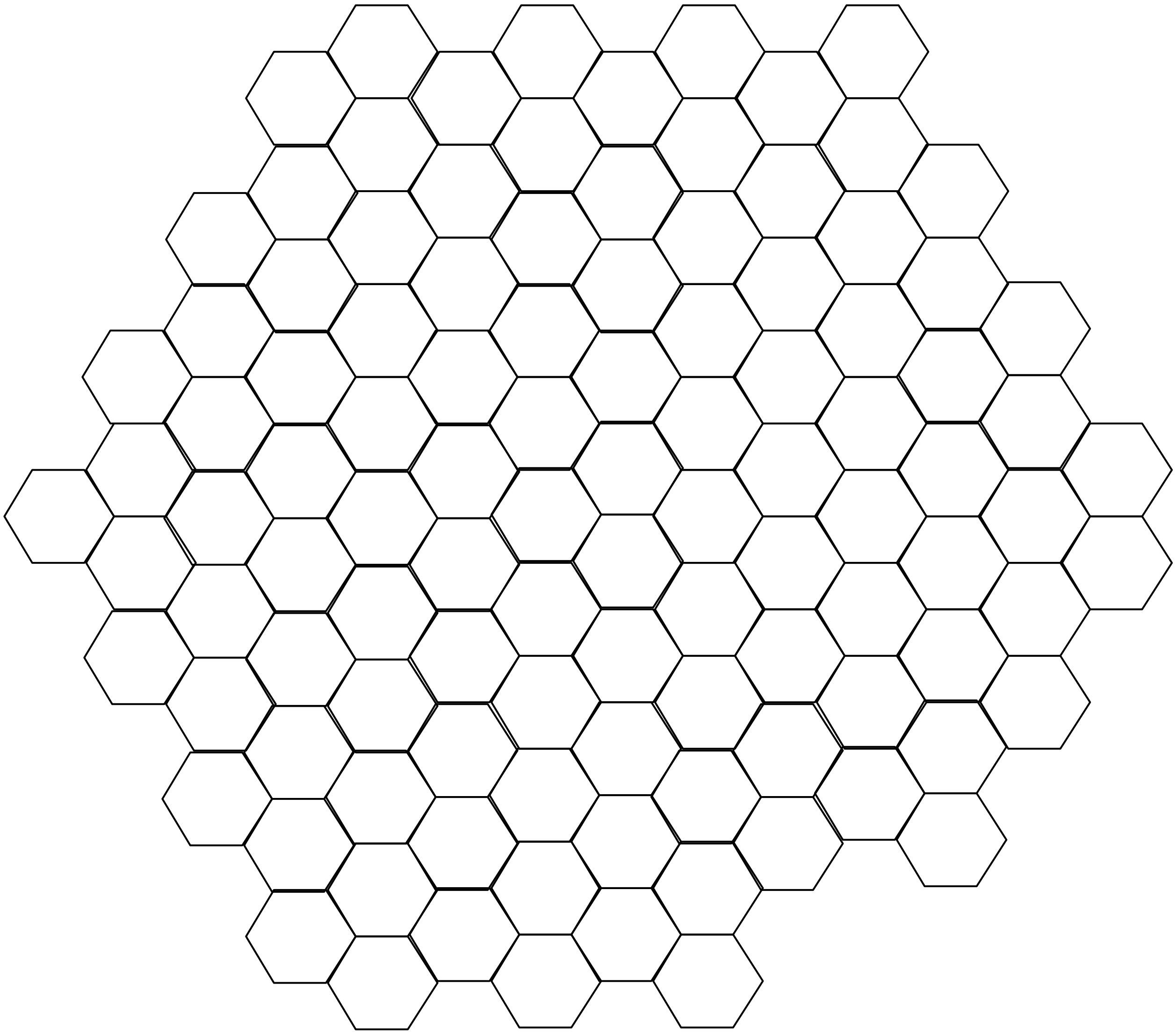
where $\beta = 3.5$. Also assume that all mobile stations are transmitting at the same power level.

a) (**3 marks:**) Show graphically the worst case co-channel interference geometry for the reverse channel. *Use the hex paper that is provided.*

b) (**4 marks:**) Calculate the worst case carrier-to-interference ratio, Λ in terms of the co-channel reuse factor D/R.

c) (**3 marks:**) Repeat parts a) and b) if 120$^{\text{o}}$ cell sectoring is used. *Use the second hex paper that is provided.*

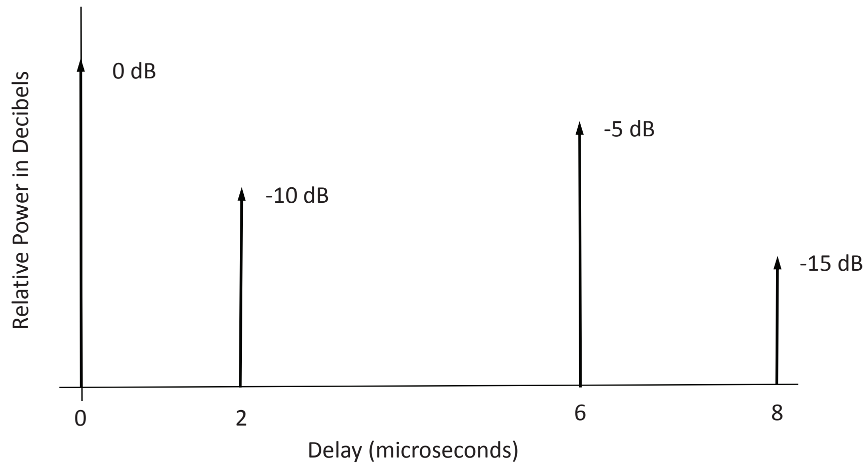




Extra sheet

Extra sheet

2) The following power-delay profile is observed for a multipath-fading channel in hilly terrain.



a) (**2 marks:**) Compute the mean delay.

b) (**3 marks:**) Compute the rms delay spread.

c) (**5 marks:**) What is the frequency correlation function of the channel?

Extra sheet

Extra sheet

3) Consider the following time-invariant channel model

$$g(t,\tau) = \frac{1}{\sqrt{L}} \sum_{k=1}^{L} \delta(\tau - (k-1)\Delta_\tau)$$

a) (**6 marks:**) Find the magnitude response $|T(f)|$ and phase response $\angle T(f)$ of the channel, where $T(f)$ is the time-invariant transfer function of the channel. Simplify your expressions as much as possible. Plot $|T(f)|$ and $\angle T(f)$.

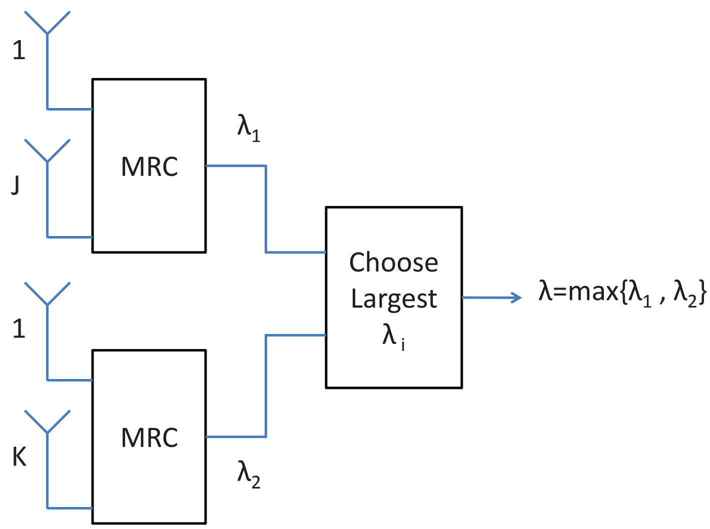
b) (**4 marks:**) What is the mean delay μ_τ and rms delay spread σ_τ of this channel? *The following may be useful:*

$$\begin{aligned}
\sum_{k=1}^{n} k &= \frac{n(n+1)}{2} \\
\sum_{k=1}^{n} k^2 &= \frac{n(n+1)(2n+1)}{6} \\
\sum_{k=0}^{n} k^3 &= \frac{n^2(n+1)^2}{4}
\end{aligned}$$

Extra sheet

Extra sheet

4) Consider a system that employs 2-branch selection diversity, where the two diversity branches consists of J and K antennas, respectively, with maximal ratio combining as shown below. Assume that all input MRC diversity branches are equal, i.e., $\bar{\gamma}_{ij} = \mathrm{E}[\gamma_{ij}] = \bar{\gamma}_c$, $i = 1, 2$, $j = 1, \ldots, J, K$.



a) 5 marks: Derive an expression for the cumulative distribution function of the symbol energy-to-noise ratio, λ, at the output of the selective combiner in terms of $\bar{\gamma}_c$, J and K.

b) 3 marks: Derive an expression for the probability density function of the symbol energy-to-noise ratio, λ, at the output of the selective combiner in terms of $\bar{\gamma}_c$, J and K.

c) 2 marks: Write down an integral expression for the probability of bit error with BPSK modulation. You do not have to solve the integral!

Extra sheet

Extra sheet

5) A guard interval consisting of a cyclic prefix or cyclic suffix is used in OFDM systems to mitigate the effects of channel time dispersion.

- **a)** (**5 points**) Assess the cost of the cyclic prefix in terms of
  - **i)** bandwidth and/or data rate.
  - **ii)** transmitter power.
- **b)** (**5 points**) Suppose the a guard interval of 500 ns is used. The data rate with 64-QAM modulation is 54 Mb/s. The power penalty due to the guard interval is to be kept less than 1 dB. What is the required value of G (constrained to an integer) and minimum the possible OFDM block size (constrained to 2^k for some k)?

Extra sheet

Extra sheet

FOURIER TRANSFORM PAIRS

Pair Number	$x(t)$	$X(f)$
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}(\tau f)$
2.	$2W \operatorname{sinc}(2Wt)$	$\Pi\left(\frac{f}{2W}\right)$
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2(\tau f)$
4.	$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$
5.	$t\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$
6.	$\exp(-\alpha\|t\|),\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$
7.	$\exp(-\alpha t^2)$	$\sqrt{\frac{\pi}{\alpha}} \exp\left(-\frac{\pi^2 f^2}{\alpha}\right)$
8.	$\delta(t)$	1
9.	1	$\delta(f)$
10.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
11.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$
12.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$
13.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$
14.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$
15.	$\operatorname{sgn}(t)$	$(j\pi f)^{-1}$
16.	$\frac{1}{\pi t}$	$-j \operatorname{sgn}(f)$
17.	$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\lambda)}{t - \lambda}\, d\lambda$	$-j \operatorname{sgn}(f) X(f)$
18.	$\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s),\ f_s = T_s^{-1}$

Note: $\operatorname{sinc} u = \frac{\sin \pi u}{\pi u}$

$$\Pi(u) = \begin{cases} 1, & \|u\| \leq 1/2 \\ 0, & \text{otherwise} \end{cases}$$

$$\Lambda(u) = \begin{cases} 1 - \|u\|, & \|u\| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

FOURIER TRANSFORM THEOREMS

Name of Theorem	Signal	Transform
1. Superposition (a_1 and a_2 arbitrary constants)	$a_1 x_1(t) + a_2 x_2(t)$	$a_1 X_1(f) + a_2 X_2(f)$
2. Time delay	$x(t - t_0)$	$X(f) \exp(-j\,2\pi f t_0)$
3a. Scale change	$x(at)$	$\|a\|^{-1} X\left(\frac{f}{a}\right)$
3b. Time reversal[1]	$x(-t)$	$X(-f) = X^*(f)$
4. Duality	$X(t)$	$x(-f)$
5a. Frequency translation	$x(t) \exp(j2\pi f_0 t)$	$X(f - f_0)$
5b. Modulation	$x(t) \cos 2\pi f_0 t$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
6. Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
7. Integration	$\int_{-\infty}^{t} x(t')\,dt'$	$(j2\pi f)^{-1} X(f) + \frac{1}{2}X(0)\delta(f)$
8. Convolution	$\int_{-\infty}^{\infty} x_1(t - t')x_2(t')\,dt'$ $= \int_{-\infty}^{\infty} x_1(t')x_2(t - t')\,dt'$	$X_1(f)X_2(f)$
9. Multiplication	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty} X_1(f - f')X_2(f')\,df'$ $= \int_{-\infty}^{\infty} X_1(f')X_2(f - f')\,df'$

[1] $x(t)$ is assumed to be real in 3b.

Pythagorean relations

$$\sin^2\alpha + \cos^2\alpha = 1, \qquad 1 + \tan^2\alpha = \sec^2\alpha, \qquad 1 + \cot^2\alpha = \csc^2\alpha$$

Angle-sum and angle-difference relations

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$
$$\cot(\alpha + \beta) = \frac{\cot\beta\cot\alpha - 1}{\cot\beta + \cot\alpha}$$
$$\cot(\alpha - \beta) = \frac{\cot\beta\cot\alpha + 1}{\cot\beta - \cot\alpha}$$
$$\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$$
$$\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha$$

Double-angle relations

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}, \qquad \cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$$

Multiple-angle relations

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$
$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$
$$\sin 4\alpha = 4\sin\alpha\cos\alpha - 8\sin^3\alpha\cos\alpha$$
$$\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$$
$$\sin 5\alpha = 5\sin\alpha - 20\sin^3\alpha + 16\sin^5\alpha$$
$$\cos 5\alpha = 16\cos^5\alpha - 20\cos^3\alpha + 5\cos\alpha$$
$$\sin 6\alpha = 32\cos^5\alpha\sin\alpha - 32\cos^3\alpha\sin\alpha + 6\cos\alpha\sin\alpha$$
$$\cos 6\alpha = 32\cos^6\alpha - 48\cos^4\alpha + 18\cos^2\alpha - 1$$
$$\sin n\alpha = 2\sin(n - 1)\alpha\cos\alpha - \sin(n - 2)\alpha$$
$$\cos n\alpha = 2\cos(n - 1)\alpha\cos\alpha - \cos(n - 2)\alpha$$
$$\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$$
$$\tan 4\alpha = \frac{4\tan\alpha - 4\tan^3\alpha}{1 - 6\tan^2\alpha + \tan^4\alpha}$$
$$\tan n\alpha = \frac{\tan(n - 1)\alpha + \tan\alpha}{1 - \tan(n - 1)\alpha\tan\alpha}$$

Function-product relations

$$\sin\alpha\sin\beta = \tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha + \beta) + \tfrac{1}{2}\sin(\alpha - \beta)$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\sin(\alpha + \beta) - \tfrac{1}{2}\sin(\alpha - \beta)$$

Function-sum and function-difference relations

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$
$$\sin\alpha - \sin\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$
$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$
$$\cos\alpha - \cos\beta = -2\sin\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$
$$\tan\alpha + \tan\beta = \frac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta}, \qquad \tan\alpha - \tan\beta = \frac{\sin(\alpha - \beta)}{\cos\alpha\cos\beta}$$
$$\cot\alpha + \cot\beta = \frac{\sin(\alpha + \beta)}{\sin\alpha\sin\beta}, \qquad \cot\alpha - \cot\beta = \frac{\sin(\beta - \alpha)}{\sin\alpha\sin\beta}$$
$$\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{\tan\frac{1}{2}(\alpha + \beta)}{\tan\frac{1}{2}(\alpha - \beta)}, \qquad \frac{\sin\alpha + \sin\beta}{\cos\alpha - \cos\beta} = \cot\tfrac{1}{2}(\beta - \alpha)$$
$$\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \tan\tfrac{1}{2}(\alpha + \beta), \qquad \frac{\sin\alpha - \sin\beta}{\cos\alpha + \cos\beta} = \tan\tfrac{1}{2}(\alpha - \beta)$$

Half-angle relations

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}, \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}} = \frac{1 - \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 + \cos\alpha}$$
$$\cot\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{1 - \cos\alpha}} = \frac{1 + \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 - \cos\alpha}$$

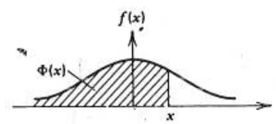
Power relations

$$\sin^2\alpha = \tfrac{1}{2}(1 - \cos 2\alpha), \qquad \sin^3\alpha = \tfrac{1}{4}(3\sin\alpha - \sin 3\alpha)$$
$$\sin^4\alpha = \tfrac{1}{8}(3 - 4\cos 2\alpha + \cos 4\alpha)$$
$$\cos^2\alpha = \tfrac{1}{2}(1 + \cos 2\alpha), \qquad \cos^3\alpha = \tfrac{1}{4}(3\cos\alpha + \cos 3\alpha)$$
$$\cos^4\alpha = \tfrac{1}{8}(3 + 4\cos 2\alpha + \cos 4\alpha)$$
$$\tan^2\alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}, \qquad \cot^2\alpha = \frac{1 + \cos 2\alpha}{1 - \cos 2\alpha}$$

Exponential relations (α in radians), Euler's equation

$$e^{i\alpha} = \cos\alpha + i\sin\alpha, \qquad i = \sqrt{-1}$$
$$\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}, \qquad \cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$
$$\tan\alpha = -i\left(\frac{e^{i\alpha} - e^{-i\alpha}}{e^{i\alpha} + e^{-i\alpha}}\right) = -i\left(\frac{e^{2i\alpha} - 1}{e^{2i\alpha} + 1}\right)$$

APPENDIX D



*Tabulation of the Standard Normal Distribution**

x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$
.00	.50000	.39894	.50	.69146	.35207	1.00	.84134	.24197
.01	.50399	.39892	.51	.69497	.35029	1.01	.84375	.23955
.02	.50798	.39886	.52	.69847	.34849	1.02	.84614	.23713
.03	.51197	.39876	.53	.70194	.34667	1.03	.84850	.23471
.04	.51595	.39862	.54	.70540	.34482	1.04	.85083	.23230
.05	.51994	.39844	.55	.70884	.34294	1.05	.85314	.22988
.06	.52392	.39822	.56	.71226	.34105	1.06	.85543	.22747
.07	.52790	.39797	.57	.71566	.33912	1.07	.85769	.22506
.08	.53188	.39767	.58	.71904	.33718	1.08	.85993	.22265
.09	.53586	.39733	.59	.72240	.33521	1.09	.86214	.22025
.10	.53983	.39695	.60	.72575	.33322	1.10	.86433	.21785
.11	.54380	.39654	.61	.72907	.33121	1.11	.86650	.21546
.12	.54776	.39608	.62	.73237	.32918	1.12	.86864	.21307
.13	.55172	.39559	.63	.73565	.32713	1.13	.87076	.21069
.14	.55567	.39505	.64	.73891	.32506	1.14	.87286	.20831
.15	.55982	.39448	.65	.74215	.32297	1.15	.87493	.20594
.16	.56356	.39387	.66	.74537	.32086	1.16	.87698	.20357
.17	.56750	.39322	.67	.74857	.31874	1.17	.87900	.20121
.18	.57142	.39253	.68	.75175	.31659	1.18	.88100	.19886
.19	.57535	.39181	.69	.75490	.31443	1.19	.88298	.19652
.20	.57926	.39104	.70	.75804	.31225	1.20	.88493	.19419
.21	.58317	.39024	.71	.76115	.31006	1.21	.88686	.19186
.22	.58706	.38940	.72	.76424	.30785	1.22	.88877	.18954
.23	.59095	.38853	.73	.76730	.30563	1.23	.89065	.18724
.24	.59484	.38762	.74	.77035	.30339	1.24	.89251	.18494
.25	.59871	.38667	.75	.77337	.30114	1.25	.89435	.18265
.26	.60257	.38568	.76	.77637	.29887	1.26	.89617	.18037
.27	.60643	.38466	.77	.77935	.29659	1.27	.89796	.17810
.28	.61026	.38361	.78	.78230	.29431	1.28	.89973	.17585
.29	.61409	.38251	.79	.78524	.29200	1.29	.90147	.17360
.30	.61791	.38139	.80	.78814	.28969	1.30	.90320	.17137
.31	.62172	.38023	.81	.79103	.28737	1.31	.90490	.16915
.32	.62552	.37903	.82	.79389	.28504	1.32	.90658	.16694
.33	.62930	.37780	.83	.79673	.28269	1.33	.90824	.16474
.34	.63307	.37654	.84	.79955	.28034	1.34	.90988	.16256
.35	.63683	.37524	.85	.80234	.27798	1.35	.91149	.16038
.36	.64058	.37391	.86	.80511	.27562	1.36	.91308	.15822
.37	.64431	.37255	.87	.80785	.27324	1.37	.91466	.15608
.38	.64803	.37115	.88	.81057	.27086	1.38	.91621	.15395
.39	.65173	.36973	.89	.81327	.26848	1.39	.91774	.15183
.40	.65542	.36827	.90	.81594	.26609	1.40	.91924	.14973
.41	.65910	.36678	.91	.81859	.26369	1.41	.92073	.14764
.42	.66276	.36526	.92	.82121	.26129	1.42	.92220	.14556
.43	.66640	.36371	.93	.82381	.25888	1.43	.92364	.14350
.44	.67003	.36213	.94	.82639	.25647	1.44	.92507	.14146
.45	.67365	.36053	.95	.82894	.25406	1.45	.92647	.13943
.46	.67724	.35889	.96	.83147	.25164	1.46	.92786	.13742
.47	.68082	.35723	.97	.83398	.24923	1.47	.92922	.13542
.48	.68439	.35553	.98	.83646	.24681	1.48	.93056	.13344
.49	.68793	.35381	.99	.83891	.24439	1.49	.93189	.13147
.50	.69146	.35207	1.00	.84134	.24197	1.50	.93319	.12952

* Abridged from *Biometrika Tables for Statisticians*, vol. 1 (2nd edition), edited by E. S. Pearson and H. O. Hartley, Cambridge University Press, London, 1958, table 1, with permission of the Biometrika Trustees.

x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$
1.50	.93319	.12952	2.00	.97725	.05399	2.50	.99379	.01753
1.51	.93448	.12758	2.01	.97778	.05292	2.51	.99396	.01709
1.52	.93574	.12566	2.02	.97831	.05186	2.52	.99413	.01667
1.53	.93699	.12376	2.03	.97882	.05082	2.53	.99430	.01625
1.54	.93822	.12188	2.04	.97932	.04980	2.54	.99446	.01585
1.55	.93943	.12001	2.05	.97982	.04879	2.55	.99461	.01545
1.56	.94062	.11816	2.06	.98030	.04780	2.56	.99477	.01506
1.57	.94179	.11632	2.07	.98077	.04682	2.57	.99492	.01468
1.58	.94295	.11450	2.08	.98124	.04586	2.58	.99506	.01431
1.59	.94408	.11270	2.09	.98169	.04491	2.59	.99520	.01394
1.60	.94520	.11092	2.10	.98214	.04398	2.60	.99534	.01358
1.61	.94630	.10915	2.11	.98257	.04307	2.61	.99547	.01323
1.62	.94738	.10741	2.12	.98300	.04217	2.62	.99560	.01289
1.63	.94845	.10567	2.13	.98341	.04128	2.63	.99573	.01256
1.64	.94950	.10396	2.14	.98382	.04041	2.64	.99585	.01223
1.65	.95053	.10226	2.15	.98422	.03955	2.65	.99598	.01191
1.66	.95154	.10059	2.16	.98461	.03871	2.66	.99609	.01160
1.67	.95254	.09893	2.17	.98500	.03788	2.67	.99621	.01130
1.68	.95352	.09728	2.18	.98537	.03706	2.68	.99632	.01100
1.69	.95449	.09566	2.19	.98574	.03626	2.69	.99643	.01071
1.70	.95543	.09405	2.20	.98610	.03547	2.70	.99653	.01042
1.71	.95637	.09246	2.21	.98645	.03470	2.71	.99664	.01014
1.72	.95728	.09089	2.22	.98679	.03394	2.72	.99674	.00987
1.73	.95818	.08933	2.23	.98713	.03319	2.73	.99683	.00961
1.74	.95907	.08780	2.24	.98745	.03246	2.74	.99693	.00935
1.75	.95994	.08628	2.25	.98778	.03174	2.75	.99702	.00909
1.76	.96080	.08478	2.26	.98809	.03103	2.76	.99711	.00885
1.77	.96164	.08329	2.27	.98840	.03034	2.77	.99720	.00861
1.78	.96246	.08183	2.28	.98870	.02965	2.78	.99728	.00837
1.79	.96327	.08038	2.29	.98899	.02898	2.79	.99736	.00814
1.80	.96407	.07895	2.30	.98928	.02833	2.80	.99744	.00792
1.81	.96485	.07754	2.31	.98956	.02768	2.81	.99752	.00770
1.82	.96562	.07614	2.32	.98983	.02705	2.82	.99760	.00748
1.83	.96638	.07477	2.33	.99010	.02643	2.83	.99767	.00727
1.84	.96712	.07341	2.34	.99036	.02582	2.84	.99774	.00707
1.85	.96784	.07206	2.35	.99061	.02522	2.85	.99781	.00687
1.86	.96856	.07074	2.36	.99086	.02463	2.86	.99788	.00668
1.87	.96926	.06943	2.37	.99111	.02406	2.87	.99795	.00649
1.88	.96995	.06814	2.38	.99134	.02349	2.88	.99801	.00631
1.89	.97062	.06687	2.39	.99158	.02294	2.89	.99807	.00613
1.90	.97128	.06562	2.40	.99180	.02239	2.90	.99813	.00595
1.91	.97193	.06438	2.41	.99202	.02186	2.91	.99819	.00578
1.92	.97257	.06316	2.42	.99224	.02134	2.92	.99825	.00562
1.93	.97320	.06195	2.43	.99245	.02083	2.93	.99831	.00545
1.94	.97381	.06077	2.44	.99266	.02033	2.94	.99836	.00530
1.95	.97441	.05959	2.45	.99286	.01984	2.95	.99841	.00514
1.96	.97500	.05844	2.46	.99305	.01936	2.96	.99846	.00499
1.97	.97558	.05730	2.47	.99324	.01889	2.97	.99851	.00485
1.98	.97615	.05618	2.48	.99343	.01842	2.98	.99856	.00471
1.99	.97670	.05508	2.49	.99361	.01797	2.99	.99861	.00457
2.00	.97725	.05399	2.50	.99379	.01753	3.00	.99865	.00443

x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$
3.00	.99865	.00443	3.50	.99977	.00087	4.00	.99997	.00013
3.01	.99869	.00430	3.51	.99978	.00084	4.01	.99997	.00013
3.02	.99874	.00417	3.52	.99978	.00081	4.02	.99997	.00012
3.03	.99878	.00405	3.53	.99979	.00079	4.03	.99997	.00012
3.04	.99882	.00393	3.54	.99980	.00076	4.04	.99997	.00011
3.05	.99886	.00381	3.55	.99981	.00073	4.05	.99997	.00011
3.06	.99889	.00370	3.56	.99981	.00071	4.06	.99998	.00011
3.07	.99893	.00358	3.57	.99982	.00068	4.07	.99998	.00010
3.08	.99897	.00348	3.58	.99983	.00066	4.08	.99998	.00010
3.09	.99900	.00337	3.59	.99983	.00063	4.09	.99998	.00009
3.10	.99903	.00327	3.60	.99984	.00061	4.10	.99998	.00009
3.11	.99906	.00317	3.61	.99985	.00059	4.11	.99998	.00009
3.12	.99910	.00307	3.62	.99985	.00057	4.12	.99998	.00008
3.13	.99913	.00298	3.63	.99986	.00055	4.13	.99998	.00008
3.14	.99916	.00288	3.64	.99986	.00053	4.14	.99998	.00008
3.15	.99918	.00279	3.65	.99987	.00051	4.15	.99998	.00007
3.16	.99921	.00271	3.66	.99987	.00049	4.16	.99998	.00007
3.17	.99924	.00262	3.67	.99988	.00047	4.17	.99998	.00007
3.18	.99926	.00254	3.68	.99988	.00046	4.18	.99999	.00006
3.19	.99929	.00246	3.69	.99989	.00044	4.19	.99999	.00006
3.20	.99931	.00238	3.70	.99989	.00042	4.20	.99999	.00006
3.21	.99934	.00231	3.71	.99990	.00041	4.21	.99999	.00006
3.22	.99936	.00224	3.72	.99990	.00039	4.22	.99999	.00005
3.23	.99938	.00216	3.73	.99990	.00038	4.23	.99999	.00005
3.24	.99940	.00210	3.74	.99991	.00037	4.24	.99999	.00005
3.25	.99942	.00203	3.75	.99991	.00035	4.25	.99999	.00005
3.26	.99944	.00196	3.76	.99992	.00034	4.26	.99999	.00005
3.27	.99946	.00190	3.77	.99992	.00033	4.27	.99999	.00004
3.28	.99948	.00184	3.78	.99992	.00031	4.28	.99999	.00004
3.29	.99950	.00178	3.79	.99992	.00030	4.29	.99999	.00004
3.30	.99952	.00172	3.80	.99993	.00029	4.30	.99999	.00004
3.31	.99953	.00167	3.81	.99993	.00028	4.31	.99999	.00004
3.32	.99955	.00161	3.82	.99993	.00027	4.32	.99999	.00004
3.33	.99957	.00156	3.83	.99994	.00026	4.33	.99999	.00003
3.34	.99958	.00151	3.84	.99994	.00025	4.34	.99999	.00003
3.35	.99960	.00146	3.85	.99994	.00024	4.35	.99999	.00003
3.36	.99961	.00141	3.86	.99994	.00023	4.36	.99999	.00003
3.37	.99962	.00136	3.87	.99995	.00022	4.37	.99999	.00003
3.38	.99964	.00132	3.88	.99995	.00021	4.38	.99999	.00003
3.39	.99965	.00127	3.89	.99995	.00021	4.39	.99999	.00003
3.40	.99966	.00123	3.90	.99995	.00020	4.40	.99999	.00002
3.41	.99968	.00119	3.91	.99995	.00019	4.41	.99999	.00002
3.42	.99969	.00115	3.92	.99996	.00018	4.42	1.00000	.00002
3.43	.99970	.00111	3.93	.99996	.00018	4.43	1.00000	.00002
3.44	.99971	.00107	3.94	.99996	.00017	4.44	1.00000	.00002
3.45	.99972	.00104	3.95	.99996	.00016	4.45	1.00000	.00002
3.46	.99973	.00100	3.96	.99996	.00016	4.46	1.00000	.00002
3.47	.99974	.00097	3.97	.99996	.00015	4.47	1.00000	.00002
3.48	.99975	.00094	3.98	.99997	.00014	4.48	1.00000	.00002
3.49	.99976	.00090	3.99	.99997	.00014	4.49	1.00000	.00002
3.50	.99977	.00087	4.00	.99997	.00013	4.50	1.00000	.00002