Georgia Institute of Technology
School of Electrical and Computer Engineering

ECE6604 Personal & Mobile Communications

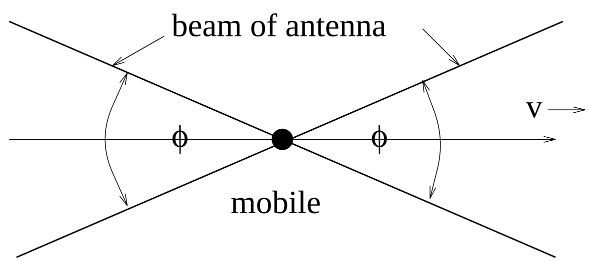
Final Exam

Fall 2018

Friday December 7, 11:20am - 2:10pm

- Attempt all questions.
- All questions are of equal value.
- Open book, open notes, exam.
- Math tables are attached at the end of this exam.
  You do not need to turn them in.

1) Consider the situation shown below where the mobile station employs a directional antenna with a beam width of ϕ^{o}. Assume a 2-D isotropic scattering environment.



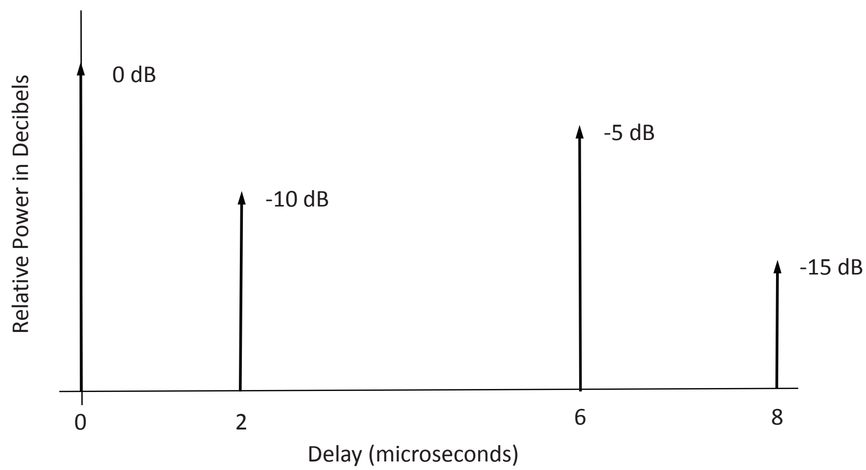
a) (**5 marks:**) In receiving a radio transmission at 850 MHz, a Doppler frequency of 20 to 60 Hz is observed. What is the beam width of the mobile station antenna, and how fast is the mobile station traveling?

b) (**5 marks:**) Suppose that the mobile station antenna has a beam width of 13^{o}. What is the level-crossing rate with respect to the rms envelope level, assuming that the mobile station is traveling at a speed of 30 km/h?

Extra sheet

Extra sheet

2) The following power-delay profile is observed for a multipath-fading channel in hilly terrain.



a) (**2 marks:**) Compute the mean delay.

b) (**3 marks:**) Compute the rms delay spread.

c) (**5 marks:**) What is the frequency correlation function of the channel?

Extra sheet

Extra sheet

3) Consider BPSK modulation on a fading channel with $L = 2$ receiver diversity.

The channel gain for Antenna 1, α_1, has the probability density function

$$p_{\alpha_1}(x) = 0.9\delta(x - 1.0) + 0.1\delta(x - 0.05) \ ,$$

while the channel gain for Antenna 2, α_2, has the probability density function

$$p_{\alpha_2}(x) = 0.7\delta(x - 1.0) + 0.3\delta(x - 0.05) \ ,$$

and α_1 and α_2 are independent.

Each diversity branch is affected by independent complex AWGN with noise power spectral density N_o watts/Hz.

Assume maximal ratio combining.

a) (**2 marks:**) What is the average bit-energy-to-noise, $\bar{\gamma}_b^{\text{mr}}$, at the output of the combiner.

b) (**4 marks:**) What is the probability density function of the bit-energy-to-noise, γ_b^{mr}, at the output of the combiner. Express your result in terms of $\bar{\gamma}_b^{\text{mr}}$.

c) (**4 marks:**) Derive an expression for the bit error probability. Express your result in terms of $\bar{\gamma}_b^{\text{mr}}$.

Extra sheet

Extra sheet

4) Consider BPSK modulation with simple binary repetition coding on a Rayleigh fading channel. Each data bit is assumed to be repeated L times, and each copy is transmitted with energy E_b/L, where E_b is the energy per data bit. Each copy is affected by independent Rayleigh fading and independent noise. The receiver is assumed to have a single antenna.

- **a) 8 marks:** At the receiver, selective combining is used to combine together the L copies of each data bit that are transmitted. Derive an expression for the probability of bit error, P_b, as a function of the average received bit energy to noise ratio $\bar{\gamma}_b$.
- **b) 2 marks:** Evaluate P_b for $L = 3$ and $\bar{\gamma}_b = 10\,\mathrm{dB}$.

Extra sheet

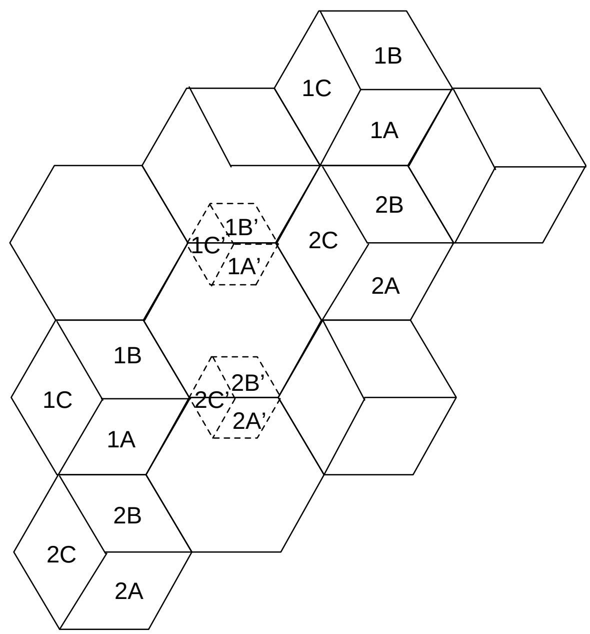
Extra sheet

5) Cell splitting is a process whereby smaller cells are introduced into an existing cellular deployment. If heavy traffic loading is experienced at the midpoint between the two cells labeled **1** in the figure below, then a split cell labeled **1'** can be introduced at that location. Likewise, the split cell labeled **2'** can be introduced at the midpoint between the two cells labeled **2**.

Assume propagation path loss according to the following model:

$$\mu_{\Omega_p}(d) = \mu_{\Omega_p}(d_o, h_{b_o}, h_{m_o}) \frac{[(h_b/h_{b_o})(h_m/h_{m_o})]^2}{(d/d_o)^\beta}$$

where $\mu_{\Omega_p\ (dBm)} = -70$ dBm at $d_o = 1\ km$, $\beta = 3.5$, $h_{b_o} = 70\ m$, $h_{m_o} = 1.5\ m$. Assume that $h_b = h_{b_o} = 70$ m and $h_m = h_{m_o} = 1.5$ m unless otherwise specified.



a) (**2 marks:**) Convert the path loss model to dBm units.

b) (**2 marks:**) If the receiver sensitivity is -115 dBm, and the transmit and receive antenna gains are unity (0 dB), what is the radius of the original cells?

c) (**2 marks:**) What is the radius of the split cells?

d) (**2 marks:**) If the MS is to receive a signal level of -115 dBm on the corner of the split cell, by how much should $\mu_{\Omega_p}(d_o, h_{b_o}, h_{m_o})$ or, equivalently, the BS transmit power be reduced or increased?

e) (**2 marks:**) Repeat part c) if the split cell uses an antenna height of only 30 m.

Extra sheet

Extra sheet

TABLE A6.3 Fourier-transform pairs

Time Function	Fourier Transform
$\text{rect}\left(\frac{t}{T}\right)$	$T\,\text{sinc}(fT)$
$\text{sinc}(2Wt)$	$\frac{1}{2W}\,\text{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a\lvert t\rvert), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{\lvert t\rvert}{T}, & \lvert t\rvert < T \\ 0, & \lvert t\rvert \geq T \end{cases}$	$T\,\text{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\text{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j\,\text{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

Notes: $u(t)$ = unit step function
$\delta(t)$ = delta function, or unit impulse
$\text{rect}(t)$ = rectangular function of unit amplitude and unit duration centered on the origin
$\text{sgn}(t)$ = signum function
$\text{sinc}(t)$ = sinc function

TABLE A6.2 Summary of properties of the Fourier transform

Property	Mathematical Description
1. Linearity	$ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where a and b are constants
2. Time scaling	$g(at) \rightleftharpoons \frac{1}{\lvert a\rvert} G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t - t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$
5. Frequency shifting	$\exp(j2\pi f_c t)g(t) \rightleftharpoons G(f - f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\,dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\,df$
8. Differentiation in the time domain	$\frac{d}{dt}g(t) \rightleftharpoons j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\,d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f) + \frac{G(0)}{2}\delta(f)$
10. Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$
11. Multiplication in the time domain	$g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2(f - \lambda)\,d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t - \tau)\,d\tau \rightleftharpoons G_1(f)G_2(f)$

Pythagorean relations

$$\sin^2\alpha + \cos^2\alpha = 1, \qquad 1 + \tan^2\alpha = \sec^2\alpha, \qquad 1 + \cot^2\alpha = \csc^2\alpha$$

Angle-sum and angle-difference relations

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$
$$\cot(\alpha + \beta) = \frac{\cot\beta\cot\alpha - 1}{\cot\beta + \cot\alpha}$$
$$\cot(\alpha - \beta) = \frac{\cot\beta\cot\alpha + 1}{\cot\beta - \cot\alpha}$$
$$\sin(\alpha + \beta)\sin(\alpha - \beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha$$
$$\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta = \cos^2\beta - \sin^2\alpha$$

Double-angle relations

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}, \qquad \cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$$

Multiple-angle relations

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$
$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$
$$\sin 4\alpha = 4\sin\alpha\cos\alpha - 8\sin^3\alpha\cos\alpha$$
$$\cos 4\alpha = 8\cos^4\alpha - 8\cos^2\alpha + 1$$
$$\sin 5\alpha = 5\sin\alpha - 20\sin^3\alpha + 16\sin^5\alpha$$
$$\cos 5\alpha = 16\cos^5\alpha - 20\cos^3\alpha + 5\cos\alpha$$
$$\sin 6\alpha = 32\cos^5\alpha\sin\alpha - 32\cos^3\alpha\sin\alpha + 6\cos\alpha\sin\alpha$$
$$\cos 6\alpha = 32\cos^6\alpha - 48\cos^4\alpha + 18\cos^2\alpha - 1$$
$$\sin n\alpha = 2\sin(n-1)\alpha\cos\alpha - \sin(n-2)\alpha$$
$$\cos n\alpha = 2\cos(n-1)\alpha\cos\alpha - \cos(n-2)\alpha$$
$$\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$$
$$\tan 4\alpha = \frac{4\tan\alpha - 4\tan^3\alpha}{1 - 6\tan^2\alpha + \tan^4\alpha}$$
$$\tan n\alpha = \frac{\tan(n-1)\alpha + \tan\alpha}{1 - \tan(n-1)\alpha\tan\alpha}$$

Formulas for Use in Trigonometry

Function-product relations

$$\sin\alpha\sin\beta = \tfrac{1}{2}\cos(\alpha - \beta) - \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}\sin(\alpha + \beta) + \tfrac{1}{2}\sin(\alpha - \beta)$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}\sin(\alpha + \beta) - \tfrac{1}{2}\sin(\alpha - \beta)$$

Function-sum and function-difference relations

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$
$$\sin\alpha - \sin\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$
$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha + \beta)\cos\tfrac{1}{2}(\alpha - \beta)$$
$$\cos\alpha - \cos\beta = -2\sin\tfrac{1}{2}(\alpha + \beta)\sin\tfrac{1}{2}(\alpha - \beta)$$
$$\tan\alpha + \tan\beta = \frac{\sin(\alpha + \beta)}{\cos\alpha\cos\beta}, \qquad \tan\alpha - \tan\beta = \frac{\sin(\alpha - \beta)}{\cos\alpha\cos\beta}$$
$$\cot\alpha + \cot\beta = \frac{\sin(\alpha + \beta)}{\sin\alpha\sin\beta}, \qquad \cot\alpha - \cot\beta = \frac{\sin(\beta - \alpha)}{\sin\alpha\sin\beta}$$
$$\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{\tan\frac{1}{2}(\alpha + \beta)}{\tan\frac{1}{2}(\alpha - \beta)}, \qquad \frac{\sin\alpha + \sin\beta}{\cos\alpha - \cos\beta} = \cot\tfrac{1}{2}(\beta - \alpha)$$
$$\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \tan\tfrac{1}{2}(\alpha + \beta), \qquad \frac{\sin\alpha - \sin\beta}{\cos\alpha + \cos\beta} = \tan\tfrac{1}{2}(\alpha - \beta)$$

Half-angle relations

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}, \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}} = \frac{1 - \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 + \cos\alpha}$$
$$\cot\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{1 - \cos\alpha}} = \frac{1 + \cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1 - \cos\alpha}$$

Power relations

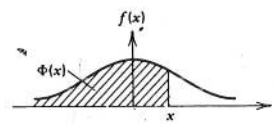
$$\sin^2\alpha = \tfrac{1}{2}(1 - \cos 2\alpha), \qquad \sin^3\alpha = \tfrac{1}{4}(3\sin\alpha - \sin 3\alpha)$$
$$\sin^4\alpha = \tfrac{1}{8}(3 - 4\cos 2\alpha + \cos 4\alpha)$$
$$\cos^2\alpha = \tfrac{1}{2}(1 + \cos 2\alpha), \qquad \cos^3\alpha = \tfrac{1}{4}(3\cos\alpha + \cos 3\alpha)$$
$$\cos^4\alpha = \tfrac{1}{8}(3 + 4\cos 2\alpha + \cos 4\alpha)$$
$$\tan^2\alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}, \qquad \cot^2\alpha = \frac{1 + \cos 2\alpha}{1 - \cos 2\alpha}$$

Exponential relations (α in radians), Euler's equation

$$e^{i\alpha} = \cos\alpha + i\sin\alpha, \qquad i = \sqrt{-1}$$
$$\sin\alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}, \qquad \cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$
$$\tan\alpha = -i\left(\frac{e^{i\alpha} - e^{-i\alpha}}{e^{i\alpha} + e^{-i\alpha}}\right) = -i\left(\frac{e^{2i\alpha} - 1}{e^{2i\alpha} + 1}\right)$$

APPENDIX D

*Tabulation of the Standard Normal Distribution**



x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$
.00	.50000	.39894	.50	.69146	.35207	1.00	.84134	.24197
.01	.50399	.39892	.51	.69497	.35029	1.01	.84375	.23955
.02	.50798	.39886	.52	.69847	.34849	1.02	.84614	.23713
.03	.51197	.39876	.53	.70194	.34667	1.03	.84850	.23471
.04	.51595	.39862	.54	.70540	.34482	1.04	.85083	.23230
.05	.51994	.39844	.55	.70884	.34294	1.05	.85314	.22988
.06	.52392	.39822	.56	.71226	.34105	1.06	.85543	.22747
.07	.52790	.39797	.57	.71566	.33912	1.07	.85769	.22506
.08	.53188	.39767	.58	.71904	.33718	1.08	.85993	.22265
.09	.53586	.39733	.59	.72240	.33521	1.09	.86214	.22025
.10	.53983	.39695	.60	.72575	.33322	1.10	.86433	.21785
.11	.54380	.39654	.61	.72907	.33121	1.11	.86650	.21546
.12	.54776	.39608	.62	.73237	.32918	1.12	.86864	.21307
.13	.55172	.39559	.63	.73565	.32713	1.13	.87076	.21069
.14	.55567	.39505	.64	.73891	.32506	1.14	.87286	.20831
.15	.55982	.39448	.65	.74215	.32297	1.15	.87493	.20594
.16	.56356	.39387	.66	.74537	.32086	1.16	.87698	.20357
.17	.56750	.39322	.67	.74857	.31874	1.17	.87900	.20121
.18	.57142	.39253	.68	.75175	.31659	1.18	.88100	.19886
.19	.57535	.39181	.69	.75490	.31443	1.19	.88298	.19652
.20	.57926	.39104	.70	.75804	.31225	1.20	.88493	.19419
.21	.58317	.39024	.71	.76115	.31006	1.21	.88686	.19186
.22	.58706	.38940	.72	.76424	.30785	1.22	.88877	.18954
.23	.59095	.38853	.73	.76730	.30563	1.23	.89065	.18724
.24	.59484	.38762	.74	.77035	.30339	1.24	.89251	.18494
.25	.59871	.38667	.75	.77337	.30114	1.25	.89435	.18265
.26	.60257	.38568	.76	.77637	.29887	1.26	.89617	.18037
.27	.60643	.38466	.77	.77935	.29659	1.27	.89796	.17810
.28	.61026	.38361	.78	.78230	.29431	1.28	.89973	.17585
.29	.61409	.38251	.79	.78524	.29200	1.29	.90147	.17360
.30	.61791	.38139	.80	.78814	.28969	1.30	.90320	.17137
.31	.62172	.38023	.81	.79103	.28737	1.31	.90490	.16915
.32	.62552	.37903	.82	.79389	.28504	1.32	.90658	.16694
.33	.62930	.37780	.83	.79673	.28269	1.33	.90824	.16474
.34	.63307	.37654	.84	.79955	.28034	1.34	.90988	.16256
.35	.63683	.37524	.85	.80234	.27798	1.35	.91149	.16038
.36	.64058	.37391	.86	.80511	.27562	1.36	.91308	.15822
.37	.64431	.37255	.87	.80785	.27324	1.37	.91466	.15608
.38	.64803	.37115	.88	.81057	.27086	1.38	.91621	.15395
.39	.65173	.36973	.89	.81327	.26848	1.39	.91774	.15183
.40	.65542	.36827	.90	.81594	.26609	1.40	.91924	.14973
.41	.65910	.36678	.91	.81859	.26369	1.41	.92073	.14764
.42	.66276	.36526	.92	.82121	.26129	1.42	.92220	.14556
.43	.66640	.36371	.93	.82381	.25888	1.43	.92364	.14350
.44	.67003	.36213	.94	.82639	.25647	1.44	.92507	.14146
.45	.67365	.36053	.95	.82894	.25406	1.45	.92647	.13943
.46	.67724	.35889	.96	.83147	.25164	1.46	.92786	.13742
.47	.68082	.35723	.97	.83398	.24923	1.47	.92922	.13542
.48	.68439	.35553	.98	.83646	.24681	1.48	.93056	.13344
.49	.68793	.35381	.99	.83891	.24439	1.49	.93189	.13147
.50	.69146	.35207	1.00	.84134	.24197	1.50	.93319	.12952

* Abridged from *Biometrika Tables for Statisticians*, vol. 1 (2nd edition), edited by E. S. Pearson and H. O. Hartley, Cambridge University Press, London, 1958, table 1, with permission of the Biometrika Trustees.

x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$
1.50	.93319	.12952	2.00	.97725	.05399	2.50	.99379	.01753
1.51	.93448	.12758	2.01	.97778	.05292	2.51	.99396	.01709
1.52	.93574	.12566	2.02	.97831	.05186	2.52	.99413	.01667
1.53	.93699	.12376	2.03	.97882	.05082	2.53	.99430	.01625
1.54	.93822	.12188	2.04	.97932	.04980	2.54	.99446	.01585
1.55	.93943	.12001	2.05	.97982	.04879	2.55	.99461	.01545
1.56	.94062	.11816	2.06	.98030	.04780	2.56	.99477	.01506
1.57	.94179	.11632	2.07	.98077	.04682	2.57	.99492	.01468
1.58	.94295	.11450	2.08	.98124	.04586	2.58	.99506	.01431
1.59	.94408	.11270	2.09	.98169	.04491	2.59	.99520	.01394
1.60	.94520	.11092	2.10	.98214	.04398	2.60	.99534	.01358
1.61	.94630	.10915	2.11	.98257	.04307	2.61	.99547	.01323
1.62	.94738	.10741	2.12	.98300	.04217	2.62	.99560	.01289
1.63	.94845	.10567	2.13	.98341	.04128	2.63	.99573	.01256
1.64	.94950	.10396	2.14	.98382	.04041	2.64	.99585	.01223
1.65	.95053	.10226	2.15	.98422	.03955	2.65	.99598	.01191
1.66	.95154	.10059	2.16	.98461	.03871	2.66	.99609	.01160
1.67	.95254	.09893	2.17	.98500	.03788	2.67	.99621	.01130
1.68	.95352	.09728	2.18	.98537	.03706	2.68	.99632	.01100
1.69	.95449	.09566	2.19	.98574	.03626	2.69	.99643	.01071
1.70	.95543	.09405	2.20	.98610	.03547	2.70	.99653	.01042
1.71	.95637	.09246	2.21	.98645	.03470	2.71	.99664	.01014
1.72	.95728	.09089	2.22	.98679	.03394	2.72	.99674	.00987
1.73	.95818	.08933	2.23	.98713	.03319	2.73	.99683	.00961
1.74	.95907	.08780	2.24	.98745	.03246	2.74	.99693	.00935
1.75	.95994	.08628	2.25	.98778	.03174	2.75	.99702	.00909
1.76	.96080	.08478	2.26	.98809	.03103	2.76	.99711	.00885
1.77	.96164	.08329	2.27	.98840	.03034	2.77	.99720	.00861
1.78	.96246	.08183	2.28	.98870	.02965	2.78	.99728	.00837
1.79	.96327	.08038	2.29	.98899	.02898	2.79	.99736	.00814
1.80	.96407	.07895	2.30	.98928	.02833	2.80	.99744	.00792
1.81	.96485	.07754	2.31	.98956	.02768	2.81	.99752	.00770
1.82	.96562	.07614	2.32	.98983	.02705	2.82	.99760	.00748
1.83	.96638	.07477	2.33	.99010	.02643	2.83	.99767	.00727
1.84	.96712	.07341	2.34	.99036	.02582	2.84	.99774	.00707
1.85	.96784	.07206	2.35	.99061	.02522	2.85	.99781	.00687
1.86	.96856	.07074	2.36	.99086	.02463	2.86	.99788	.00668
1.87	.96926	.06943	2.37	.99111	.02406	2.87	.99795	.00649
1.88	.96995	.06814	2.38	.99134	.02349	2.88	.99801	.00631
1.89	.97062	.06687	2.39	.99158	.02294	2.89	.99807	.00613
1.90	.97128	.06562	2.40	.99180	.02239	2.90	.99813	.00595
1.91	.97193	.06438	2.41	.99202	.02186	2.91	.99819	.00578
1.92	.97257	.06316	2.42	.99224	.02134	2.92	.99825	.00562
1.93	.97320	.06195	2.43	.99245	.02083	2.93	.99831	.00545
1.94	.97381	.06077	2.44	.99266	.02033	2.94	.99836	.00530
1.95	.97441	.05959	2.45	.99286	.01984	2.95	.99841	.00514
1.96	.97500	.05844	2.46	.99305	.01936	2.96	.99846	.00499
1.97	.97558	.05730	2.47	.99324	.01889	2.97	.99851	.00485
1.98	.97615	.05618	2.48	.99343	.01842	2.98	.99856	.00471
1.99	.97670	.05508	2.49	.99361	.01797	2.99	.99861	.00457
2.00	.97725	.05399	2.50	.99379	.01753	3.00	.99865	.00443

x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$	x	$\Phi(x)$	$f(x)$
3.00	.99865	.00443	3.50	.99977	.00087	4.00	.99997	.00013
3.01	.99869	.00430	3.51	.99978	.00084	4.01	.99997	.00013
3.02	.99874	.00417	3.52	.99978	.00081	4.02	.99997	.00012
3.03	.99878	.00405	3.53	.99979	.00079	4.03	.99997	.00012
3.04	.99882	.00393	3.54	.99980	.00076	4.04	.99997	.00011
3.05	.99886	.00381	3.55	.99981	.00073	4.05	.99997	.00011
3.06	.99889	.00370	3.56	.99981	.00071	4.06	.99998	.00011
3.07	.99893	.00358	3.57	.99982	.00068	4.07	.99998	.00010
3.08	.99897	.00348	3.58	.99983	.00066	4.08	.99998	.00010
3.09	.99900	.00337	3.59	.99983	.00063	4.09	.99998	.00009
3.10	.99903	.00327	3.60	.99984	.00061	4.10	.99998	.00009
3.11	.99906	.00317	3.61	.99985	.00059	4.11	.99998	.00009
3.12	.99910	.00307	3.62	.99985	.00057	4.12	.99998	.00008
3.13	.99913	.00298	3.63	.99986	.00055	4.13	.99998	.00008
3.14	.99916	.00288	3.64	.99986	.00053	4.14	.99998	.00008
3.15	.99918	.00279	3.65	.99987	.00051	4.15	.99998	.00007
3.16	.99921	.00271	3.66	.99987	.00049	4.16	.99998	.00007
3.17	.99924	.00262	3.67	.99988	.00047	4.17	.99998	.00007
3.18	.99926	.00254	3.68	.99988	.00046	4.18	.99999	.00006
3.19	.99929	.00246	3.69	.99989	.00044	4.19	.99999	.00006
3.20	.99931	.00238	3.70	.99989	.00042	4.20	.99999	.00006
3.21	.99934	.00231	3.71	.99990	.00041	4.21	.99999	.00006
3.22	.99936	.00224	3.72	.99990	.00039	4.22	.99999	.00005
3.23	.99938	.00216	3.73	.99990	.00038	4.23	.99999	.00005
3.24	.99940	.00210	3.74	.99991	.00037	4.24	.99999	.00005
3.25	.99942	.00203	3.75	.99991	.00035	4.25	.99999	.00005
3.26	.99944	.00196	3.76	.99992	.00034	4.26	.99999	.00005
3.27	.99946	.00190	3.77	.99992	.00033	4.27	.99999	.00004
3.28	.99948	.00184	3.78	.99992	.00031	4.28	.99999	.00004
3.29	.99950	.00178	3.79	.99992	.00030	4.29	.99999	.00004
3.30	.99952	.00172	3.80	.99993	.00029	4.30	.99999	.00004
3.31	.99953	.00167	3.81	.99993	.00028	4.31	.99999	.00004
3.32	.99955	.00161	3.82	.99993	.00027	4.32	.99999	.00004
3.33	.99957	.00156	3.83	.99994	.00026	4.33	.99999	.00003
3.34	.99958	.00151	3.84	.99994	.00025	4.34	.99999	.00003
3.35	.99960	.00146	3.85	.99994	.00024	4.35	.99999	.00003
3.36	.99961	.00141	3.86	.99994	.00023	4.36	.99999	.00003
3.37	.99962	.00136	3.87	.99995	.00022	4.37	.99999	.00003
3.38	.99964	.00132	3.88	.99995	.00021	4.38	.99999	.00003
3.39	.99965	.00127	3.89	.99995	.00021	4.39	.99999	.00003
3.40	.99966	.00123	3.90	.99995	.00020	4.40	.99999	.00002
3.41	.99968	.00119	3.91	.99995	.00019	4.41	.99999	.00002
3.42	.99969	.00115	3.92	.99996	.00018	4.42	1.00000	.00002
3.43	.99970	.00111	3.93	.99996	.00018	4.43	1.00000	.00002
3.44	.99971	.00107	3.94	.99996	.00017	4.44	1.00000	.00002
3.45	.99972	.00104	3.95	.99996	.00016	4.45	1.00000	.00002
3.46	.99973	.00100	3.96	.99996	.00016	4.46	1.00000	.00002
3.47	.99974	.00097	3.97	.99996	.00015	4.47	1.00000	.00002
3.48	.99975	.00094	3.98	.99997	.00014	4.48	1.00000	.00002
3.49	.99976	.00090	3.99	.99997	.00014	4.49	1.00000	.00002
3.50	.99977	.00087	4.00	.99997	.00013	4.50	1.00000	.00002