

TABLE A6.3 *Fourier-transform pairs*

Time Function	Fourier Transform
$\text{rect}\left(\frac{t}{T}\right)$	$T \text{sinc}(fT)$
$\text{sinc}(2Wt)$	$\frac{1}{2W} \text{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a t), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{ t }{T}, & t < T \\ 0, & t \geq T \end{cases}$	$T \text{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\text{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j \text{sgn}(f)$
$u(t)$	$\frac{1}{2} \delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

Notes: $u(t)$ = unit step function $\delta(t)$ = delta function, or unit impulse $\text{rect}(t)$ = rectangular function of unit amplitude and unit duration centered on the origin $\text{sgn}(t)$ = signum function $\text{sinc}(t)$ = sinc function**TABLE A6.2** *Summary of properties of the Fourier transform*

Property	Mathematical Description
1. Linearity	$ag_1(t) + bg_2(t) \Rightarrow aG_1(f) + bG_2(f)$ where a and b are constants
2. Time scaling	$g(at) \Rightarrow \frac{1}{ a } G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \Rightarrow G(f)$, then $G(t) \Rightarrow g(-f)$
4. Time shifting	$g(t - t_0) \Rightarrow G(f) \exp(-j2\pi f t_0)$
5. Frequency shifting	$\exp(j2\pi f_c t)g(t) \Rightarrow G(f - f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t) dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f) df$
8. Differentiation in the time domain	$\frac{d}{dt} g(t) \Rightarrow j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^t g(\tau) d\tau \Rightarrow \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2} \delta(f)$
10. Conjugate functions	If $g(t) \Rightarrow G(f)$, then $g^*(t) \Rightarrow G^*(-f)$
11. Multiplication in the time domain	$g_1(t)g_2(t) \Rightarrow \int_{-\infty}^{\infty} G_1(\lambda)G_2(f - \lambda) d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t - \tau) d\tau \Rightarrow G_1(f)G_2(f)$