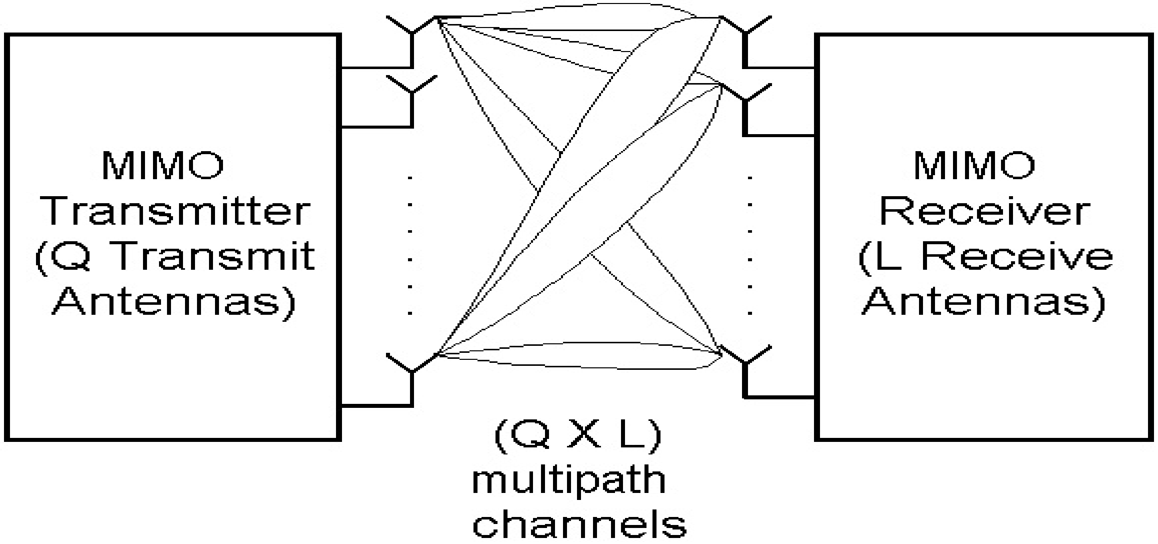
Lecture 29

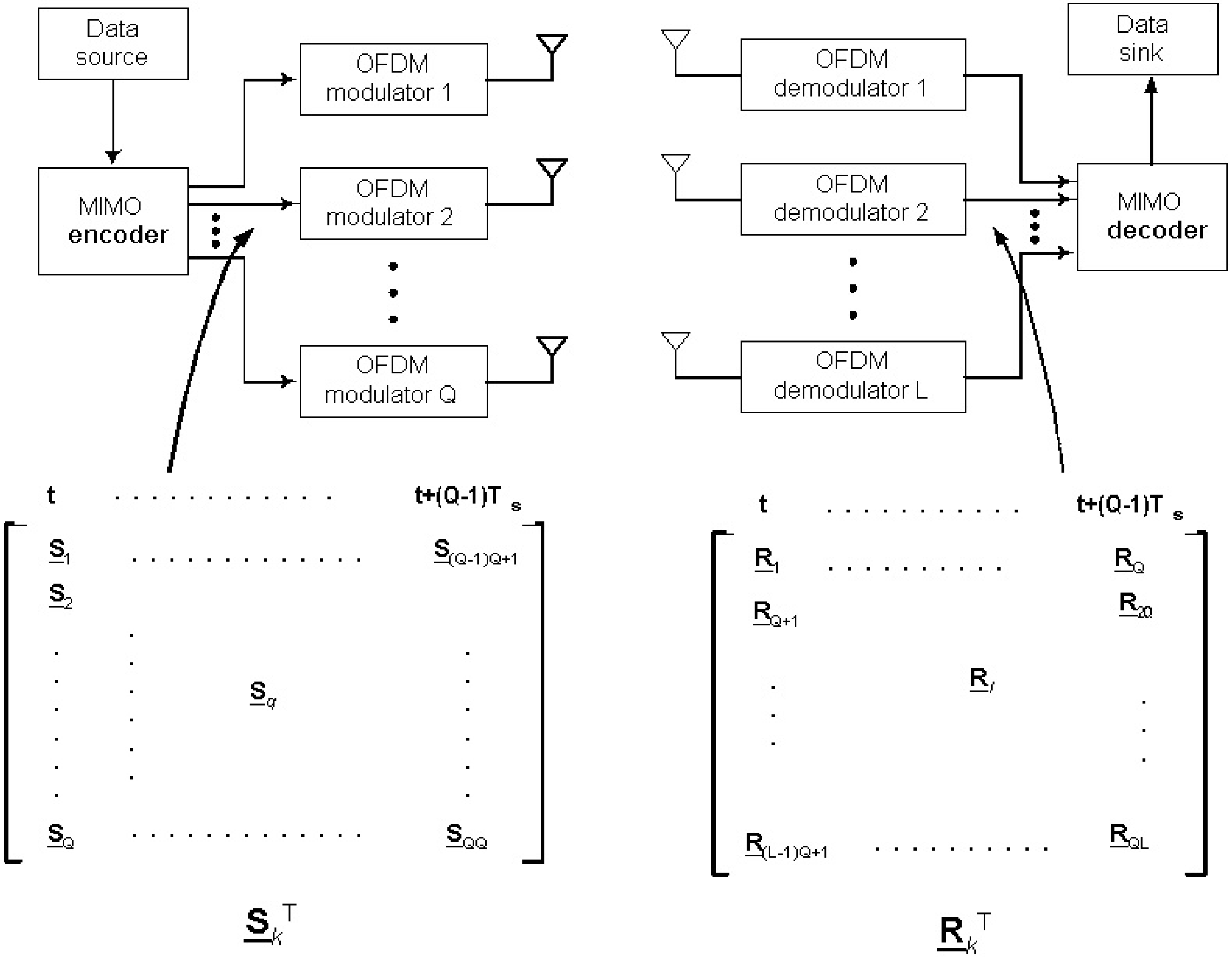
OFDM Synchronization

A MULTI INPUT MULTI OUTPUT (MIMO) OFDM SYSTEM



- **A MIMO system uses Q Transmit antennas and L Receive Antennas**

Q-TRANSMIT L-RECEIVE MIMO OFDM SYSTEM



$$\begin{array}{cccc} t & \cdots & \cdots & t+(Q-1)T_s \\ \left[\begin{array}{c} \underline{S}_1 \\ \underline{S}_2 \\ \vdots \\ \vdots \\ \underline{S}_Q \end{array}\right. & \begin{array}{c} \cdots \\ \vdots \\ \underline{S}_q \\ \vdots \\ \cdots \end{array} & & \left.\begin{array}{c} \underline{S}_{(Q-1)Q+1} \\ \\ \vdots \\ \vdots \\ \underline{S}_{QQ} \end{array}\right] \end{array}$$

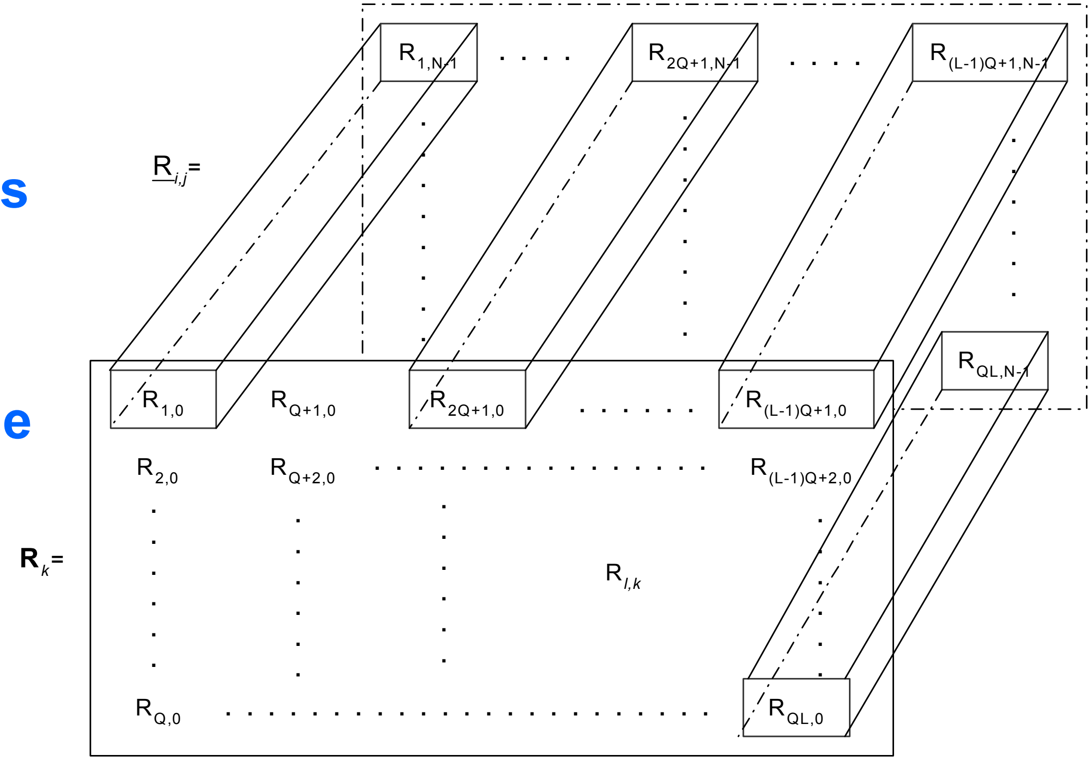
$\underline{S}_k^T$

$$\begin{array}{cccc} t & \cdots & \cdots & t+(Q-1)T_s \\ \left[\begin{array}{c} \underline{R}_1 \\ \underline{R}_{Q+1} \\ \vdots \\ \vdots \\ \underline{R}_{(L-1)Q+1} \end{array}\right. & \begin{array}{c} \cdots \\ \\ \underline{R}_l \\ \\ \cdots \end{array} & & \left.\begin{array}{c} \underline{R}_Q \\ \underline{R}_{2Q} \\ \vdots \\ \vdots \\ \underline{R}_{QL} \end{array}\right] \end{array}$$

$\underline{R}_k^T$

SYSTEM EQUATION

- **The received demodulated OFDM sample matrix R can be expressed in terms of the transmitted sample matrix S, the channel coefficient matrix η and the noise matrix W as:**



d=OFDM symbol

q=TX antenna

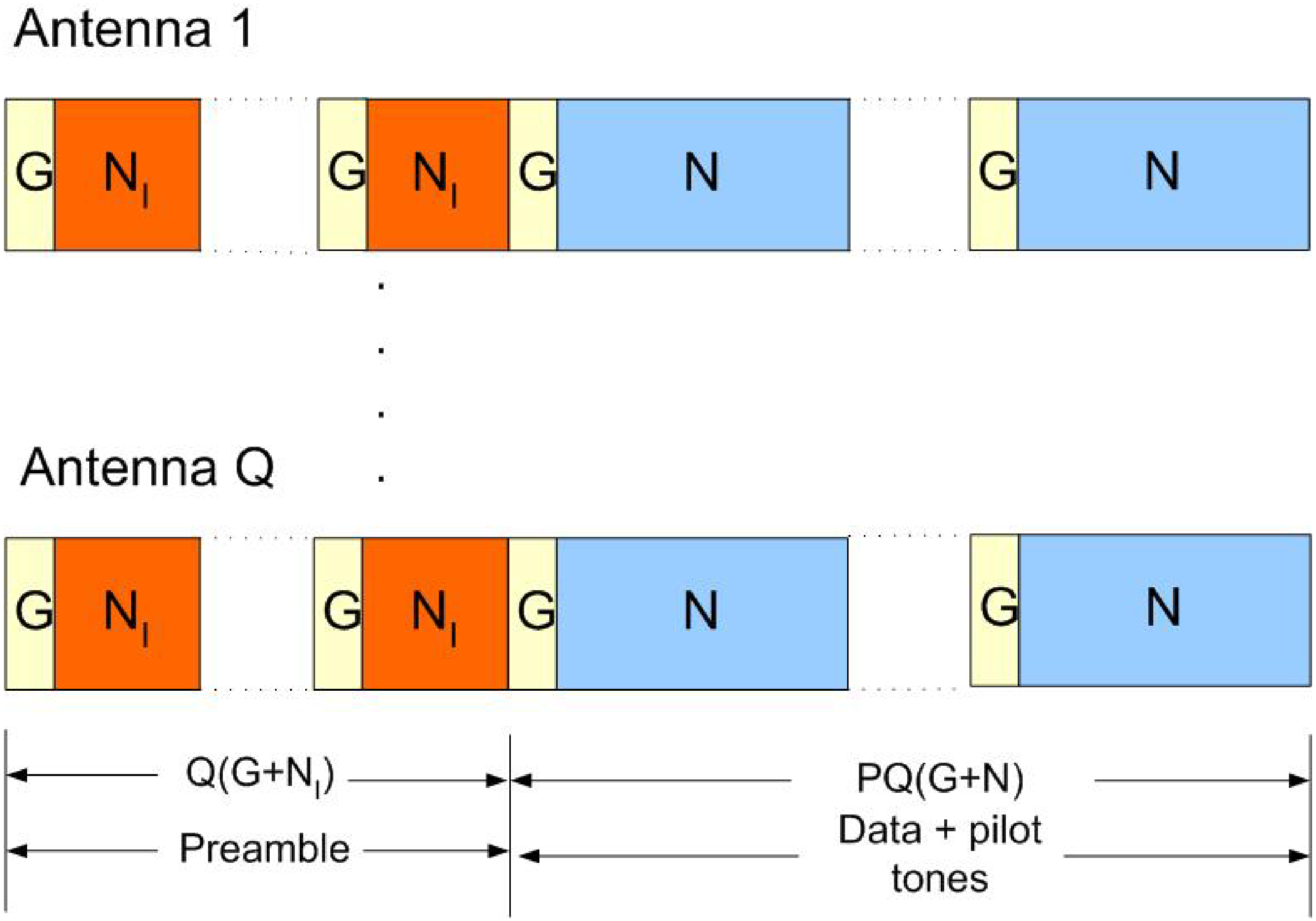
l=RX antenna

k=subcarrier

$$r_{ij}[n,k]$$

$$R_{d,l,k}=\sum_{q=1}^{Q}\exp\left\{j\frac{2\pi}{N}\big(dk\beta(N+G)\big)+\frac{\gamma}{2}(N-1)\right\}\mathrm{sinc}(\beta k)\,\mathrm{sinc}(\gamma)\,\eta_{q,l,k}S_{q,k}+W_{d,l,k,AWGN}+W_{d,l,k,ICI}$$

GENERAL FRAME STRUCTURE FOR A MIMO OFDM SYSTEM



MIMO OFDM FRAME CONSTRUCTION

- Preamble consists of Q OFDM symbols of a generalized length N_l, where $N_l=N/l$, $l=1,2,4$ etc.
- Data symbols consist of P blocks of Q OFDM symbols having length N
- Each symbol is preceded by a cyclic prefix of G samples.
- The preamble sequences of length N_l can be constructed by
  - exciting every lth subchannel of an N point sequence in the frequency domain using some known alphabet,

MIMO OFDM FRAME CONSTRUCTION (Contnd.)

- Taking an N-point IFFT of the sequence,
- Keep the first N_I samples and discarding the rest,
- Add a cyclic prefix to the sequence before transmission.

- Hence the training sequence for the qth symbol in the time domain is given by

$$s_{q,n} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_{q,k} \exp\left(\frac{j2\pi nk}{N}\right) \qquad n = 0,1,\ldots,N_I - 1.$$

CHARACTERISTICS OF GOOD PREAMBLE SEQUENCES AND STRUCTURES

- Good correlation properties for time synchronization
- Low PAPR for high power transmission
- Suitable for channel parameter estimation
- Suitable for frequency offset estimation over a wide range
- Low computational complexity, low overhead but high accuracy

GENERATION OF LENGTH 256 SEQUENCE N=256, l=1

Example: For N_l=256

S1=sqrt(2)*[0 1 -1 1 -1 -1 1 -1 1 -1 1 1 1 -1 1 1 -1 -1 1 -1
1 -1 1 -1 1 1 1 -1 -1 -1 1 -1 1 -1 1 1 -1 1 -1 -1 1 -1 -1
-1 -1 1 -1 1 1 1 1 1 -1 -1 -1 1 -1 1 -1 -1 -1 -1 1 -1 -1
-1 1 -1 -1 1 -1 1 1 1 -1 1 -1 1 -1 -1 1 1 -1 -1 1 -1 -1 -1
-1 1 1 1 1 1 -1 -1 -1 -1 1 1 -1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1
-1 1 -1 -1 -1 -1 1 1 -1 1 -1 -1 1 -1 1 1 -1 1 -1 -1 1 1
1 1 -1 1 -1 1 1 -1 1 -1 -1 -1 -1 -1 -1 1 -1 1 1 1 1 1 1
-1 1 1 -1 -1 1 1 1 -1 -1 1 -1 1 1 -1 -1 1 -1 1 1 1 -1
-1 1 1 1 1 1 -1 1 -1 1 -1 1 1 1 1 -1 -1 -1 1 -1 -1 1 1
1 1 1 -1 -1 -1 1 1 -1] PAPR = 5.34 dB

55 0’s come from IEEE802.16a spectral requirements

GENERATION OF LENGTH 128 SEQUENCE N=256, l=2

Example: For N_l=128

S1=sqrt(2)*[0 0 -1 0 -1 0 1 0 -1 0 -1 0 1 0 1 0 1
0 -1 0 1 0 1 0 1 0 -1 0 -1 0 -1 0 -1 0 -1 0 -1
0 1 0 -1 0 -1 0 -1 0 -1 0 -1 0 -1 0 1 0 1 0 1
0 -1 0 1 0 -1 0 1 0 1 0 -1 0 1 0 1 0 1 0 -1
0 -1 0 -1 0 -1 0 -1 0 1 0 -1 0 -1 0 1 0 -1 0 -1
0 1 0 -1 {55 0’s} -1 0 1 0 1 0 1 0 1 0 -1 0 -1 0
1 0 -1 0 1 0 -1 0 -1 0 1 0 1 0 -1 0 1 0 -1 0 1
0 -1 0 1 0 -1 0 1 0 1 0 -1 0 1 0 -1 0 -1 0 1
0 -1 0 -1 0 -1 0 1 0 1 0 -1 0 1 0 1 0 1 0 -1
0 1 0 1 0 -1 0 -1 0 -1 0 1 0 1 0 1 0 1 0 1 0
1 0 1 0]

PAPR = 4.31 dB

GENERATION OF LENGTH 64 SEQUENCE N=256, I=4

Example: For $N_I=64$

S1=sqrt(2)*[0 0 0 0 +1+j 0 0 0 -1-j 0 0 0 +1+j 0 0 0 +1-j 0 0 0 +1+j 0 0 0 +1+j 0 0 0 -1+j 0 0 0 +1+j 0 0 0 -1-j 0 0 0 +1+j 0 0 0 +1+j 0 0 0 +1-j 0 0 0 +1+j 0 0 0 +1+j 0 0 0 +1-j 0 0 0 -1-j 0 0 0 +1+j 0 0 0 -1-j 0 0 0 -1-j 0 0 0 -1+j 0 0 0 +1+j 0 0 0 +1+j 0 0 0 +1-j 0 0 0 -1-j 0 0 0 -1+j 0 0 0

-1+j {55 0's} +1+j 0 0 0 +1+j 0 0 0 +1+j 0 0 0 -1+j 0 0 0 +1+j 0 0 0 -1-j 0 0 0 -1+j 0 0 0 -1-j 0 0 0 -1-j 0 0 0 -1-j 0 0 0 +1+j 0 0 0 -1+j 0 0 0 +1+j 0 0 0 -1-j 0 0 0 +1-j 0 0 0 +1+j 0 0 0 +1+j 0 0 0 +1+j 0 0 0 -1-j 0 0 0 +1-j 0 0 0 +1+j 0 0 0 -1-j 0 0 0 +1-j 0 0 0 +1+j 0 0 0 -1+j 0 0 0 +1-j 0 0 0]

PAPR = 3.00 dB

OFDM SIGNAL ACQUISITION USING PREAMBLE

The preamble at the start of an OFDM frame is used to acquire the OFDM signal and perform:

- Time synchronization
  - Coarse time synchronization – Step I
  - Fine time synchronization – Step IV
- Frequency offset estimation
  - Fractional frequency offset estimation – Step II
  - Residual frequency offset estimation - Step III
- Channel and noise variance estimation

OFDM SIGNAL ACQUISITION

Step I. Coarse Time Synchronization –

- Estimate the start of the OFDM frame over an approximate range of samples. It must be robust.
- Techniques – Perform maximum-likelihood estimation of the time-of-arrival
  - The likelihood function is approximated by [van de Beek]

$$\Lambda(n,\gamma) \approx |\phi_n| \cos\left(\frac{2\pi\gamma}{I} + \angle\phi_n \right)$$

Where γ is the frequency offset between Tx and Rx local oscillators and φ_n is given by

$$\phi_n = \sum_{k=0}^{G-1} \left(r^*_{j,n+k} \cdot r_{j,n+k+N_I} \right)$$

Frequency Offset Estimation, Step II

Step II. Fractional Frequency Offset Estimation

- **Extremely important since frequency offset introduces ICI,**
- **Technique – Maximum-likelihood estimation of the frequency offset**

$$\hat{\gamma}_{\mathrm{ML}} = \arg\max_{\gamma}\left(\Lambda\left(d_{\mathrm{opt}}, \gamma\right)\right)$$

- **The function is maximized when the cosine in the likelihood function is maximum. Hence,**

$$\hat{\gamma}_{\mathrm{ML}} = \frac{I}{2\pi} \cdot \angle \phi_{d_{\mathrm{opt}}}$$

Residual Frequency Offset Estimation, Step III

- The range of the maximum-likelihood frequency offset estimator is ± I / 2 subchannel spacing.

- This frequency offset estimation/ correction range can be improved using some frequency domain processing.

Step III. Residual Frequency Offset Estimation

- If the same sequence $s_{i,n}$, $n=0,\ldots,N_I-1$ is transmitted from all the antennas then the frequency offset of integral multiples of subchannel spacing can be carried out.

Residual Frequency Offset Estimation, Step III

- Sequence $s_{i,n}$, and the received frequency corrected samples

$$r_{1,n}^{c} = r_{1,n} \exp\left\{ j2\pi \hat{\gamma}_{\mathrm{ML}} n / N \right\}$$

corresponding to the preamble for $n=0,1,\ldots,N_I-1$ are repeated I times and passed through an N-point FFT to obtain $S_{i,n}$ and $R_{1,n}$.

- Periodic cross-correlation of the received demodulated OFDM symbol $R_{1,n}$ with $S_{i,n}$ is carried out as

$$\chi_k = \sum_{n=0}^{N-1} S^{*}_{i,(k+n)_N} R^{c}{}_{1,n} \qquad k = 0,1,\ldots,N-1$$

Residual Frequency Offset Estimation, Step III

- The residual frequency offset of an integral number of subchannel spacing is obtained as

$$\hat{\Gamma} = \arg\max_{k} \left\{ \left| \chi_k \right| \right\}$$

- The residual frequency offset estimate $\hat{\Gamma}$ can be sent to the local oscillator (NCO) for offset correction.

Fine Time Synchronization, Step IV

Step IV. Fine Time Synchronization

- Fine time synchronization is needed to obtain start of the OFDM frame to within a few samples,
- It can be carried out by cross-correlating the received frequency offset corrected samples with the transmitted sequence as

$$\psi_n = \sum_{i=1}^{Q} \left| \sum_{k=0}^{N-1} s_{i,k}^{*} r_{j,n+k}^{c} \right| \;, \quad j = 1, \ldots, L$$

- If same sequence is transmitted from all the antennas then only one cross-correlator is needed.

PARAMETER ESTIMATION

Channel Estimation for MIMO OFDM Systems

- Step I. –
  - –LS Estimation using Q symbols

$$\eta_k = \mathrm{S}_k^H \mathrm{R}_k = \mathrm{S}_k^{-1} \mathrm{R}_k \qquad k = 0, I\ldots., N_I I - 1.$$

PARAMETER ESTIMATION

- Step II. – Interpolation in the Frequency Domain

  Channel estimates are needed for all the tones, however, they are available for only N_l tones. If channel statistics are not available at the receiver then frequency domain (linear) interpolation may be used otherwise MMSE interpolation may be used.

COMMERCIAL OFDM SYSTEMS

- In commercial OFDM systems, the tone at d.c. and the tones near the band-edges are set to zero.

- This is called zero-padding, or subchannel nulling and the zero-padded tones are called virtual subchannels.

- For example in IEEE 802.16a/b Broadband Fixed Wireless Access systems, out of N=256, 56 tones are set to zero. Hence the number of tones used N_u=200.

- Before employing Method I for MSE reduction, frequency domain extrapolation is needed.

MSE REDUCTION IN FREQUENCY DOMAIN

- MSE reduction can be carried out in the frequency domain itself. One of the simplest methods is frequency domain smoothing.
- Keep the tones from the coarse channel estimates near the band-edges as they are and perform averaging on all the other tones using

$$\hat{\eta}_{ij,k} = \frac{\overline{\eta}_{ij,k-1} + \overline{\eta}_{ij,k+1}}{2}$$

SIGNAL TRANSMISSION MATRIX DESIGN

- Need unitary S_ks in order to generate Q OFDM symbols of a generalized length N_l for channel estimation.
- The simplest unitary structure is obtained when the signal transmission matrix is diagonal
  - Direct extension of SISO
  - The transmitted power needs to be increased by a factor of Q in the training phase. Hence, it requires power amplifiers with an increased dynamic range.

$$S_D = \begin{bmatrix} \underline{S}_1 & 0 & 0 & 0 \\ 0 & \underline{S}_2 & 0 & 0 \\ 0 & 0 & \underline{S}_3 & 0 \\ 0 & 0 & 0 & \underline{S}_4 \end{bmatrix}$$

SIGNAL TRANSMISSION MATRIX DESIGN

- For Q=2, Alamouti' s structure is optimal

$$\mathrm{S}_{A} = \begin{bmatrix} \underline{S}_1 & \underline{S}_2 \\ \underline{-S}_2^* & \underline{S}_1^* \end{bmatrix} \qquad \mathrm{S}_{AS} = \begin{bmatrix} \underline{S}_1 & \underline{S}_1 \\ \underline{-S}_1^* & \underline{S}_1^* \end{bmatrix}$$

- For Q=4 and 8, orthogonal signal sets can be used, e.g. for Q=4,

$$\mathrm{S}_{TS} = \begin{bmatrix} \underline{S}_1 & \underline{S}_1 & \underline{S}_1 & \underline{S}_1 \\ \underline{-S}_1 & \underline{S}_1 & \underline{-S}_1 & \underline{S}_1 \\ \underline{-S}_1 & \underline{S}_1 & \underline{S}_1 & \underline{-S}_1 \\ \underline{-S}_1 & \underline{-S}_1 & \underline{S}_1 & \underline{S}_1 \end{bmatrix}$$

SIMULATION RESULTS FOR SIGNAL ACQUISITION

- Simulations for the system performance are carried out for an IEEE802.16a Broadband Fixed Wireless Access System.
- The fixed wireless access channel is characterized by the Stanford University Interim (SUI) models.
- SUI-4 Channel Model for moderate to heavy tree densities is given by:

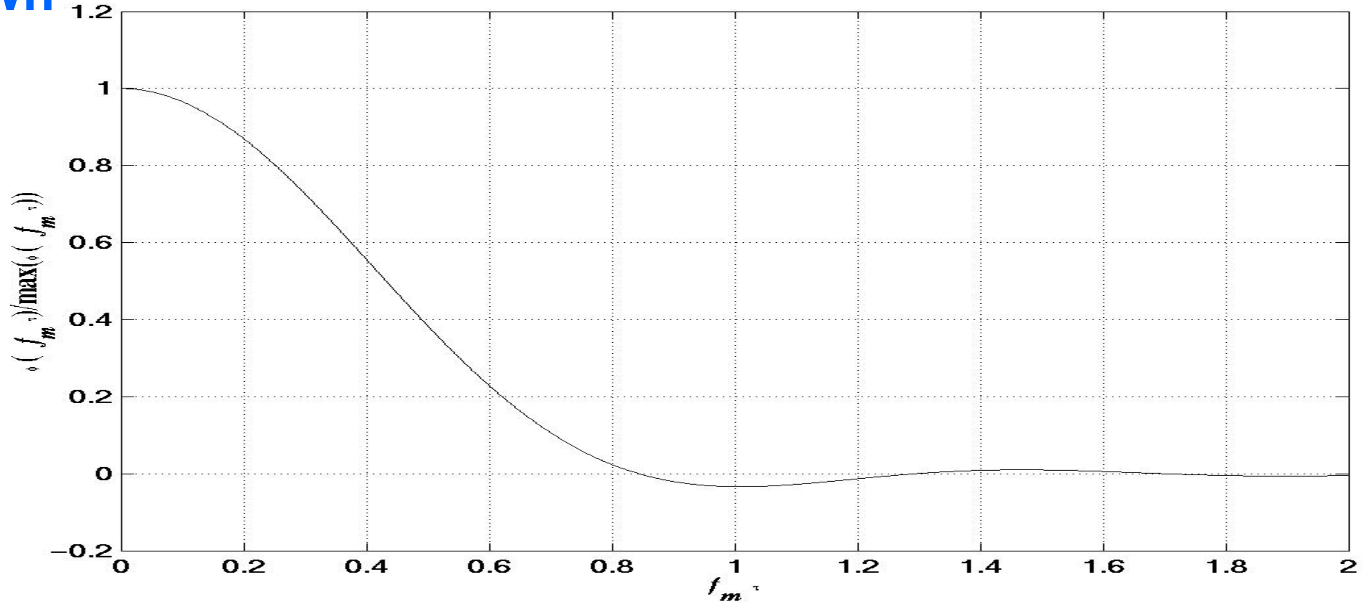
	Tap1	Tap2	Tap3	Units
Delay	0	1.5	4	μS
Power	0	-4	-8	dB
f_m	0.2	0.15	0.25	Hz

SIMULATION RESULTS FOR SIGNAL ACQUISITION

- The Doppler power spectrum for the SUI channel taps is approximated by

$$S(f)=\begin{cases}1-1.72f_0^2+0.784f_0^4 & f_0\le 1\\ 0 & f_0>1\end{cases}\qquad f_0=\frac{f}{f_m}$$

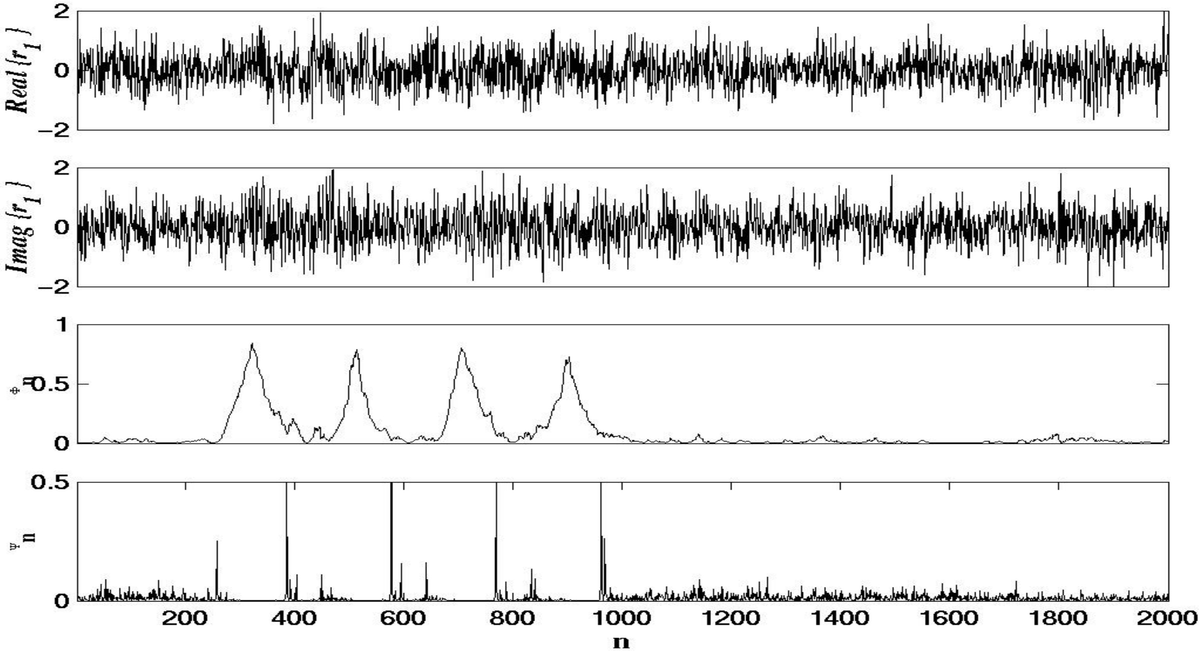
- Autocorrelation Function and PSD for a SUI Channel Tap are as shown



SIMULATION RESULTS

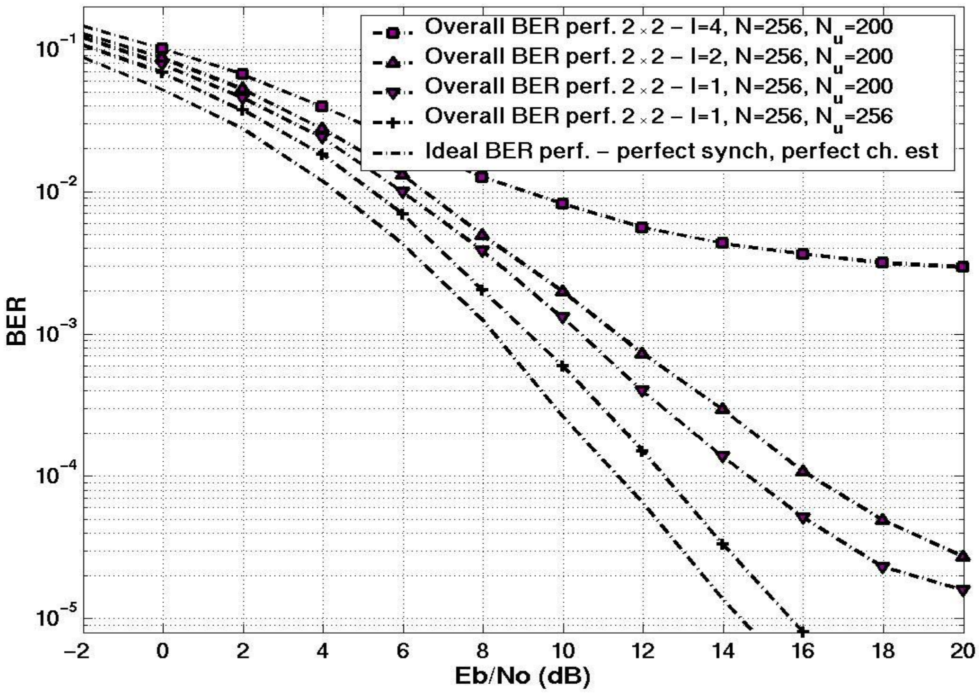
- Bandwidth = 3.5 MHz, Block size N = 256, Guard G = 64, Modulation type – 16-QAM, P=Number of space-time blocks per frame=10, No channel coding employed,
- Rate 1 space-time block code (STBC) used for a 2X2 system and rate ¾ STBC used for a 4X4 system.
- Total frequency offset $\Gamma+\gamma$ = 1+0.25 subchannel spacing,
- Number of tones used, N_u=200,
- Training sequences used are those proposed for IEEE802.16a.

TIME SYNCHRONIZATION



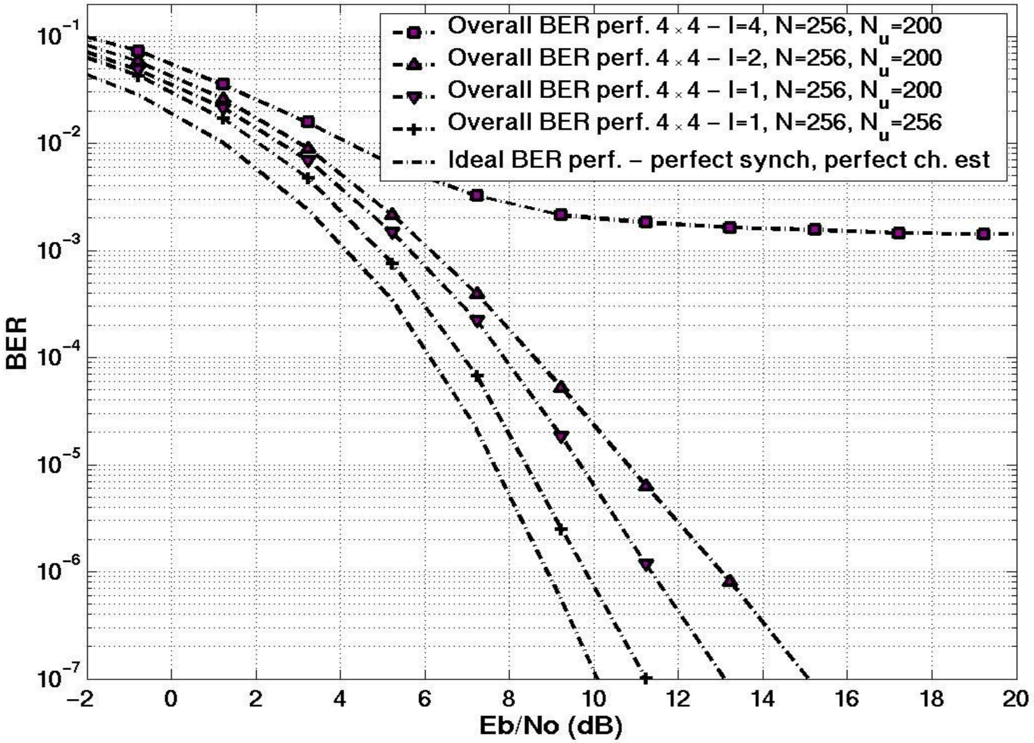
Coarse and fine time synchronization for a 4X4 system with N_I=128, SNR of 10 dB and frequency offset 1.25 subchannel spacing. Steps I. and IV.

BER PERMANCE FOR A 2X2 SYSTEM



Uncoded BER as a function of SNR for a 2X2 system using 16-QAM modulation and after synchronization and channel estimation.

BER PERMANCE FOR A 4X4 SYSTEM



Uncoded BER as a function of SNR for a 4X4 system using 16-QAM modulation and after synchronization and channel estimation.